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EuroGebra - KA229 Project 2018/21



EUROGEBRA WORKSHEETS

Plane geometry I



EUROGEBRA WORKSHEETS

TITLE: PLANE GEOMETRY

Introduction:

These worksheets were created within the Erasmus + project, Eurogebra.

Worksheets are in the field of mathematics and use the Geogebra program for individual mathematical tasks. The purpose is to use the program when teaching and explaining problems in mathematics and thus to approach the issue more clearly.

Worksheets are in the form of specific instructions and tools that will guide us to solve various tasks. In this way, students will get closer to a better understanding and modification of the given examples. Individual groups of worksheets can be combined with each other and create new subgroups according to the issues discussed. Some examples are followed by the solution of examples and free sheets for student notes.

Worksheets respect pedagogical documents related to the subject of mathematics. When working with worksheets, it is necessary to cooperate with teachers and coordinate their work.

In terms of content, we focused on geometric examples, where you can effectively solve problems and modify them in various ways. By formulating the tasks, we lead the students to understand the assigned tasks and to solve the tasks according to the individual steps in the worksheets.



EUROGEBRA WORKSHEET

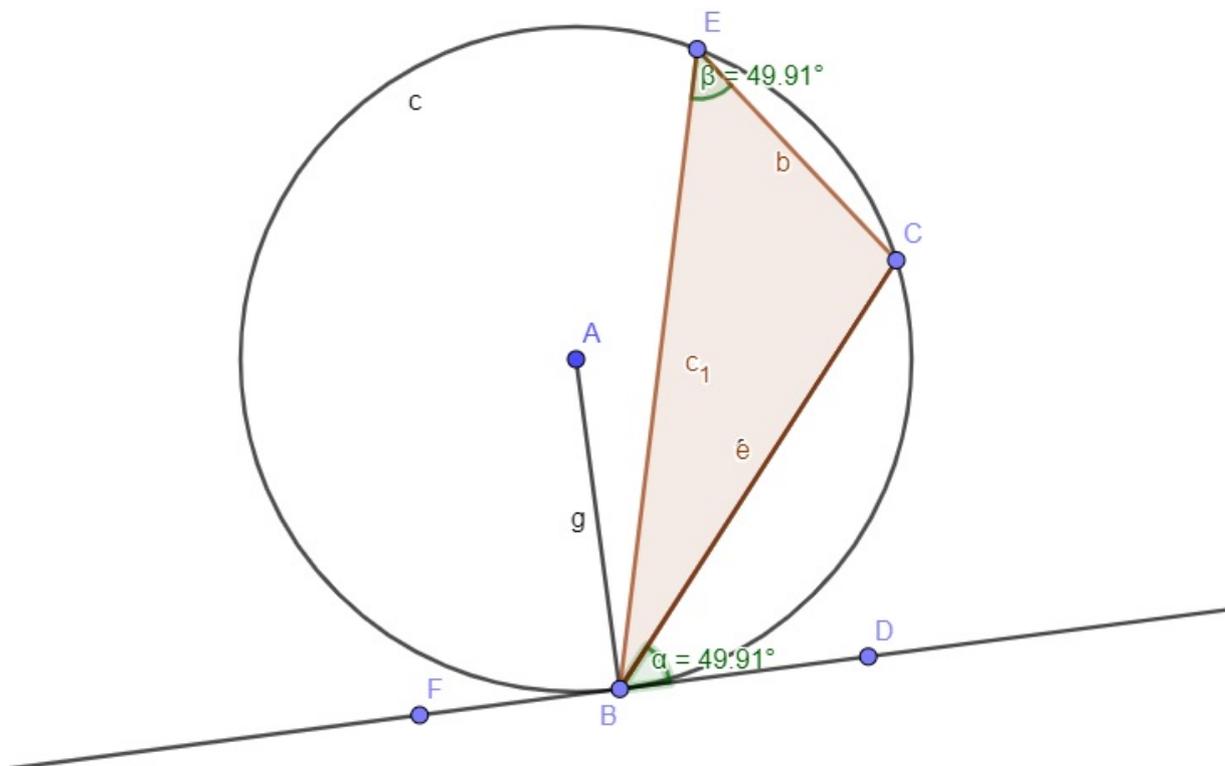
ANGLE WITH TANGENT

MENU	TOOL	PROCESS STEPS
	 Circle with Centre and Radius	Draw a circle with centre A and given radius - for example: 4
	 Point on Object	Create points B and C somewhere on the circle
	 Segment	Draw a segment between points B and C
	 Segment	Draw a segment between points A and B
	 Perpendicular Line	Draw a line perpendicular to segment AB , going through the point B
	 Point on Object	Create a point D on the line drawn in the previous step, on that side of segment AB where the point C is.
	 Angle	Insert an angle between points D, B and C . In settings choose <i>Angle between: 0° and 180°</i> .
	 Point on Object	Create a point E somewhere on the circle
	 Polygon	Draw a polygon B, E, C



	 Angle	Insert an angle between points B , E and C . In settings choose <i>Angle between: 0° and 180°</i> .
	 Point on Object	Insert a point F on the tangent but NOT on the ray BD
	 Angle	Insert an angle between points F , B and C . In settings choose <i>Angle between: 0° and 180°</i> .
		In settings of α angle go to <i>advanced</i> tab and in <i>Conditions to Show Object</i> type in $\alpha = \beta$
		In settings of γ angle go to <i>advanced</i> tab and in <i>Conditions to Show Object</i> type in $\alpha \neq \beta$

Angle between a tangent and a chord of a circle





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EUROGEBRA WORKSHEET

CENTRE OF A CIRCLE

Find the centre of a circle given three points on that circle

MENU	TOOL	PROCESS STEPS
	 Circle through 3 Points	Construct a circle through three points
	 Segment	Draw a line segment that passes through point A and B (chord AB)
	 Segment	Draw a line segment that passes through point B and C (chord BC)
	 Perpendicular Bisector	Construct the perpendicular bisector of AB
	 Perpendicular Bisector	Construct the perpendicular bisector of BC
	 Intersect	Define the point D formed by the intersection of the two perpendicular bisectors (point D is the centre of the circle)

It works by joining two pairs of points to create two chords. The perpendicular bisector of each chord always passes through the center of the circle. By this method we find the centre.



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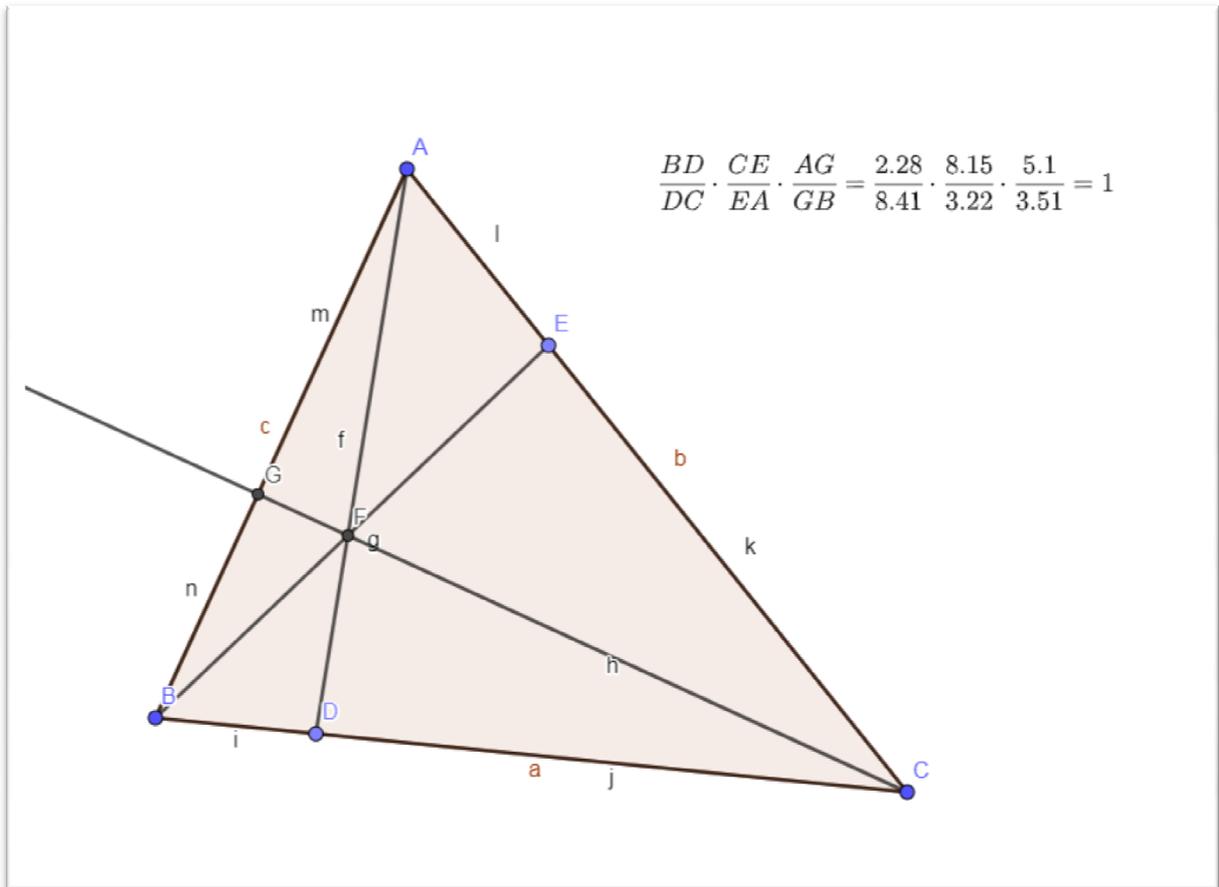
EUROGEBRA WORKSHEET

CEVA THEOREM

MENU	TOOL	PROCESS STEPS
 Polygon	Construct the triangle ABC	
 Point	<ul style="list-style-type: none"> • Create point D on the segment BC • Create point E on the segment AC 	
 Segment	<ul style="list-style-type: none"> • Create the segment AD • Create the segment BE 	
 Intersect	Create the intersection point F from segments AD and BE	
 Ray	Create the CF ray	
 Intersect	Create the intersection point G from segments AB and ray CF	
 Segment	<ul style="list-style-type: none"> • Create segment BD (=i) • Create segment DC (=j) • Create segment EC (=k) • Create segment EA (=l) • Create segmentGA (=m) • Create segmentGB (=n) 	
	Go to Algebra section and type $\frac{i}{j} \cdot \frac{k}{\ell} \cdot \frac{m}{n} (=d)$	



	<p>Go to Geometry section and Text button and type:</p> <p>Text</p> <p>B I Serif LaTeX formula</p> <p>$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AG}{GB} = \frac{f}{i} \cdot \frac{k}{l} \cdot \frac{m}{n} = d$</p> <p>Advanced</p> <p>Preview edit LaTeX formula</p> <p>(empty box)</p> <p>$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AG}{GB} = \frac{f}{i} \cdot \frac{k}{l} \cdot \frac{m}{n} = d$</p> <p>OK CANCEL</p>	
	<p>Can you find a relation to the Menelaos Theorem?</p>	

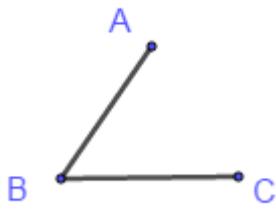




EUROGEBRA WORKSHEET

LINE SEGMENT

DRAW A LINE (SEGMENT) THAT BISECTS THE GIVEN ANGLE

MENU	TOOL	PROCESS STEPS
		
	 Circle with Centre through Point	Draw a circle with center B
	 Intersect	Define the points that formed with intersection of circle and segments(E and F points)
	 Compasses	Draw two circles with radius equal to length BE or BF and E and F-centered
	 Intersect	Define the points that formed with intersection of circles that E and F-centered (G points)
	 Line	Connect G and B points with the "Line" tool.
	 Angle	Check the angles CBG and GBA



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EUROGEBRA WORKSHEET

LINE SEGMENT

MENU	TOOL	PROCESS STEPS
	 Line	Draw a line. Pick arbitrary points A and B on this line.
	 Point on Object	
	 Point	Get a C point on the line (not between A and B)
	 Compasses	Construct a circle with center C with radius equal to length of AB
	 Intersect	Define the point that formed with intersection of circle and AB segment with the intersection tool (D point)
	 Distance or Length	Check the line segments AB and CD



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EUROGEBRA WORKSHEET

LINE THROUGH POINTS

DRAW A LINE (SEGMENT) PERPENDICULAR TO THE GIVEN LINE GOING THROUGH POINT C

MENU	TOOL	PROCESS STEPS
	Circle with Centre through Point	Draw the circle with the center point C and cutting the line at two points.
	Intersect	Define the points that formed with intersection of circle and line with the intersection tool (E and F points)
	Compasses	Draw two circles with radius equal to length EC or FC and center D and E
	Intersect	Define the points that formed with intersection of D and E circles with the intersection tool (C and G points)
	Segment	Connect C and G points with the "Segment" tool
	Intersect	Define the points that formed with intersection of segments with the intersection tool (H point)
	Angle	Check the angle CHE or FHC



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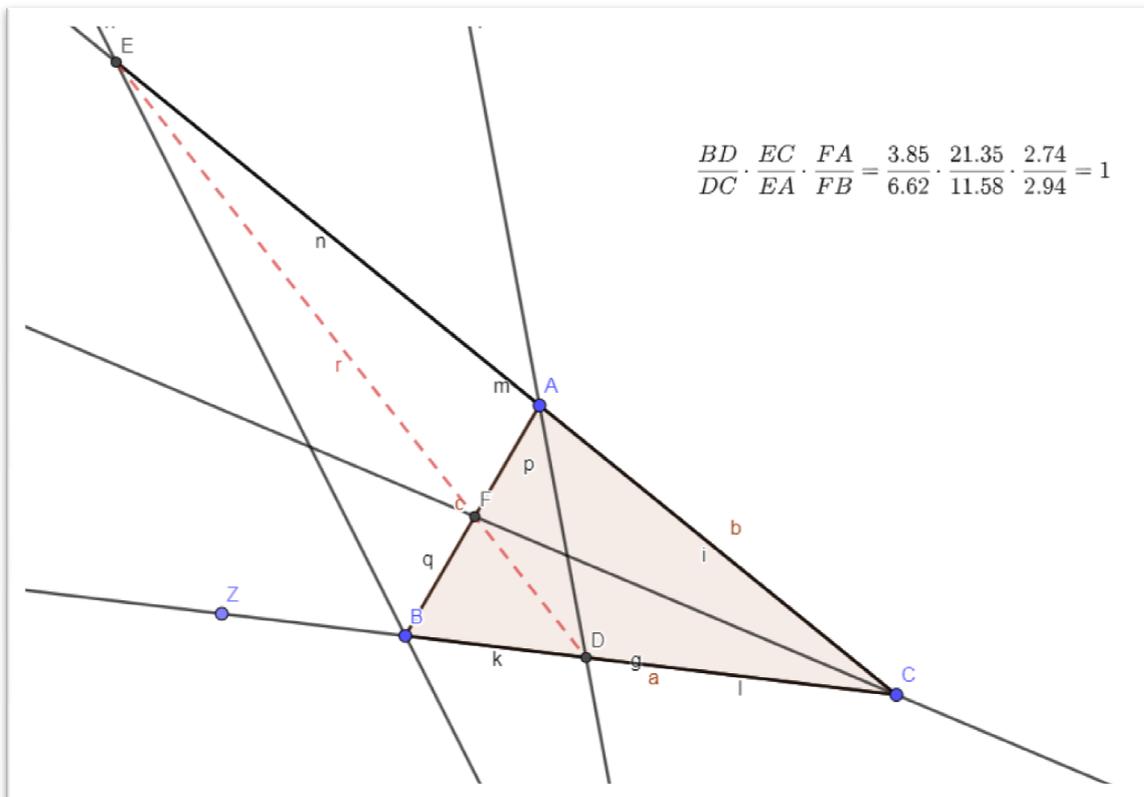
EUROGEBRA WORKSHEET

MENELAOS THEOREM

MENU	TOOL	PROCESS STEPS
 Polygon	Construct the triangle ABC	
 Angle Bisector	Create the angle bisector of the angle A	
 Intersect	Create the intersection point D from Bisector A and segment(BC)	
 Ray	Create the CB ray and then the point Z on it, outside the segment BC	
 Angle Bisector	Create the bisector of the angle ZBA.	
 Ray	Create the CA ray	
 Intersect	Create the intersect point E between ray CA and bisector of the ZBA angle	
 Angle Bisector	Create the bisector of the angle BAC.	
 Intersect	Create the intersect point F between segment AB and bisector of the BAC angle	



 Segment	<ul style="list-style-type: none"> • Create segment BD (=k) • Create segment DC (=l) • Create segment EC (=m) • Create segment EA (=n) • Create segment FA (=p) • Create segment FB (=q) 	
	<p>Go to Algebra section and type $\frac{k}{l} \cdot \frac{m}{n} \cdot \frac{p}{q} (=d)$</p>	
 	<p>Go to Geometry section and Text button and type:</p> <p>Text</p> <p>$\frac{BD}{DC} \cdot \frac{EC}{EA} \cdot \frac{FA}{FB} = \frac{k}{l} \cdot \frac{m}{n} \cdot \frac{p}{q} = d$</p> <p>$\frac{BD}{DC} \cdot \frac{EC}{EA} \cdot \frac{FA}{FB} = \frac{k}{l} \cdot \frac{m}{n} \cdot \frac{p}{q} = d$</p>	
	<p>What you can say as a result about the position of the points E,F,D ?</p>	





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POINTS IN LINES

MENU	TOOL	PROCESS STEPS
	 Line	Draw a line that pass through point A (AC line)
	 Parallel Line	Draw a parallel line to AC that pass through point B (BD line)
	 Circle with Centre and Radius	Construct a circle with radius equal to 3 and center A
	 Circle with Centre and Radius	Construct a circle with radius equal to 5 and center B
	 Intersect	Define the points that formed with intersection of circles and lines with the intersect tool (E and F points)
	 Line	Combine E and F points with the “Line” tool.
	 Intersect	Define the points that formed with intersection of AB and EF lines with the intersect tool (G point)
	 Distance or Length	Check the length GA and GB line segments



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EUROGEBRA WORKSHEET

TRIANGLE AREAS

Compare triangle areas with the same base and different apex point

MENU	TOOL	PROCESS STEPS
	Segment	Draw a segment
	Point	Draw a point outside these segments line
	Parallel Line	Click the point and the segment to draw a parallel line to the segment
	Polygon	Successively select the three points which will be the vertices of the triangle. Then, click the first point again in order to close the triangle.
	Area	Point the triangle shape and you get the area of the triangle
	Point	Draw a point on the line, different to the first point
	Polygon	Successively select the new point on the line and the other two points at the base and the new point again to create another triangle
	Area	Click the shape of the new triangle to get the area
	Move	Click on the last point to change the shape of the last triangle. What did you see?



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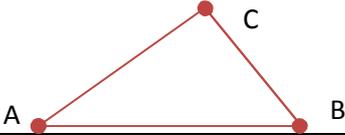




EUROGEBRA WORKSHEET

TRIANGLE

Find the circumcentre and the circumcircle of a triangle

MENU	TOOL	PROCESS STEPS
		
	 Circle with Centre and Radius	Construct a circle with radius equal to AB and centre A
	 Circle with Centre and Radius	Construct a circle with radius equal to AB and centre B
	 Intersect	Define the points formed by the intersection the two circles (D and E points)
	 Line	Connect D and E points with the “Line” tool.
	 Circle with Centre and Radius	Construct a circle with radius equal to AC and centre A
	 Circle with Centre and Radius	Construct a circle with radius equal to AC and centre C
	 Intersect	Define the points formed by the intersection the two circles (F and G points)
	 Line	Connect F and G points with the “Line” tool.



	 Intersect	<p>Define the point formed by the intersection the two lines (point H). This is the circumcentre.</p>
	 Circle with Centre and Radius	<p>Construct a circle with radius equal to AH and center H. This is the circumcircle.</p>