



Plane geometry II





TITLE: PLANE GEOMETRY

Introduction:

These worksheets were created within the Erasmus + project, Eurogebra.

Worksheets are in the field of mathematics and use the Geogebra program for individual mathematical tasks. The purpose is to use the program when teaching and explaining problems in mathematics and thus to approach the issue more clearly.

Worksheets are in the form of specific instructions and tools that will guide us to solve various tasks. In this way, students will get closer to a better understanding and modification of the given examples. Individual groups of worksheets can be combined with each other and create new subgroups according to the issues discussed. Some examples are followed by the solution of examples and free sheets for student notes.

Worksheets respect pedagogical documents related to the subject of mathematics. When working with worksheets, it is necessary to cooperate with teachers and coordinate their work.

In terms of content, we focused on geometric examples, where you can effectively solve problems and modify them in various ways. By formulating the tasks, we lead the students to understand the assigned tasks and to solve the tasks according to the individual steps in the worksheets.



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EUROGEBRA WORKSHEET

CENTER OF CIRCLE

FIND THE CENTER OF CIRCLE

MENU	TOOL	PROCESS STEPS
\odot	Circle through 3 Points	Draw circle that through A,B,C points
_**	✓ Segment	Combine the A and B points
\odot	Compass	Draw a circle that radius equal to length of AB segment and centered at C
• ^A	Intersect	Define the points that formed with intersections of circles(D point)
*	Segment	Combine the C and D points
÷	Perpendicular Bisector	Draw perpendicular bisectors of AB and CD segments
• ^A	Intersect	Define the points that formed with intersections of perpendicular bisectors (Center of circle)
4	Distance or Length	Measure from center point to a point which is on the circle









DIVIDING A SEGMENT

Dividing a segment into 3 pieces of equal length

MENU	TOOL	PROCESS STEPS
A A	Segment	Draw [AB] line segment
**	Ray	Draw a ray starting in A
a=2	$\frac{a=2}{\bullet}$ Slider	Create a slider <i>a</i> =1 MIN 1 MAX 10 INTERCEPT 0.1
\odot	 Circle with Centre and Radius 	Draw a circle with radius <i>a</i> and a center at A
•	Intersect	Find the intersection of the circle with the ray. Mark it as point D
$\overline{oldsymbol{\circ}}$	 Circle with Centre and Radius 	Draw a circle with radius <i>a</i> and a center at D.
•	Intersect	Find the intersection of the circle with the ray. Mark it as point E
\bigcirc	 Circle with Centre and Radius 	Draw a circle with radius <i>a</i> and a center at E.





• ^A	Intersect	Find the intersection of the circle and the ray. Mark it as point F.
x x	Line	Create a line through points F and B
- T	Parallel Line	Draw lines parallel to the one from the previous step, that go through points E and D respectively
•	Intersect	Find the intersections of these lines with segment AB. Mark them as I and J.
- Ar Ar	Segment	Drew segments [AI], [IJ] and [JB]
α	Distance or Length	Check the length of these segments





EQUAL ANGLE

DRAW AN ANGLE EQUAL TO THE GIVEN ANGLE

MENU	TOOL	PROCESS STEPS
		B
\odot	• Circle with Centre through Point	Draw the circle with the center point B and cutting the segments at two points.
• ^A	Intersect	Define the points that formed with intersection of circle and line with the intersection tool (E and F points)
•	, Line	Draw a new line (GH line)
•	Compasses	Drawa circle with radiusequal to length BEand center it at G
• ^A	Intersect	Define the point that formed with intersection of circle and line with the intersection tool (I point)
	Compasses	Drawa circle with radiusequal to lengthEFand center it at I
•	Intersect	Define the points that formed with intersection of circles (J point)





A	✓ Segment	Connect G and J points with the "Segment" tool.
4	\land Angle	Check the angles CBA and IGJ





FIND THE INCENTRE AND THE INSCRIBED CIRCLE OF A TRIANGLE

MENU	TOOL	PROCESS STEPS
		C B
+	Angle Bisector	Constructthe angle bisector of angle BAC.
-	Angle Bisector	Construct the angle bisector of angle ABC.
•	Intersect	Find the intersection of the two angle bisectors (point D). This is the incentre.
ł	Perpendicular Line	Construct a line perpendicular to the line AB that goes through point D.
•	Intersect	Find the intersection of the perpendicular line and the line AB (point E).
\bigcirc	• Circle with Center through Point	Construct a circle with centre at point D and radius DE. This is the inscribed circle.





DRAW AN ISOSCELES TRIANGLE

MENU	TOOL	PROCESS STEPS
	🦯 Segment	Draw an arbitrary segment AB
*	→ Perpendicular Bisector	Draw perpendicular bisector of segment AB
• •	• ^A Point	Define arbitrary point C on perpendicular bisector
	🥕 Segment	Draw segments by selecting points A, C and B, C
	⊳ Polygon	Select points A,B,C to draw triangle
	Distance or Length	Check and compare lenght of segments AB and BC
	Angle	Check and compare angles between segments AB, AC and BC, AB
	l⊋ Move	Move point C and watch changes. What can we say?











CONSTRUCT A LINE WITH LENGHT $\sqrt{12}$

MENU	TOOL	PROCESS STEPS
		We find two numbers whose product is 12 and whose length we can construct. For example, 4 and 3.
	🥍 Úsečka s danou dĺžkou	Create a line A, B with a length of 7cm (4 + 3)
	🥍 Úsečka s danou dĺžkou	Mark the point C on the line AB at a distance of 4 cm from the point A
-+	Kolmica	Make a perpendicular line to line AB in point C
• ^A	• Stred	Locate the midpoint of the line AB - point D
\bigcirc	• Kružnica daná stredom a bodom	Make a circle with the center D and the radius DB, respectively DA
• ^A	Priesečník	Determine the intersection of the circle and the perpendicular to the line AB - point F
4	Vzdialenosť alebo dĺžka	Check the length of the FC line

The solution is based on Euclid's theorem of height: v2 = ca. cbIn this case (FC) 2 = AC.CB $h = \sqrt{12}$ cm, ca = 4cm, cb = 3cm









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EUROGEBRA WORKSHEET

FIND THE MEDIAN POINT OF TRIANGLE

MENU	TOOL	PROCESS STEPS
	Polygon	Create an arbitrary triangle ABC
• ^A	Midpoint or Centre	Determine the midpoint of segment AB – point D
	🦯 Segment	Connect point D with opposed vertex C
• ^A	Midpoint or Centre	Determine the midpoint of segment BC – point E
	🦯 Segment	Connect point E with opposed vertex A.
• ^A	Intersect	Determine intersection of segments AE and DC – point F.
	Distance or Length	Compare the lengths of segments AF, FE and DF, FC. What's their ratio?
	G Move	Move any vertex and watch the position of median point.









MIDPOINT OF THE LINE

DRAWTHE MIDPOINT OF THE LINE SEGMENT

MENU	TOOL	PROCESS STEPS
- Carl	Segment	Draw [AB] line segment
\odot	• Circle with Centre through Point	Draw two circles that with radius equal to length of AB and to is centered it at A and B
•	Intersect	Define the points that formed with intersection of circles with the intersection tool (C and D points)
and a	Segment	Connect two points with the "Segment" tool.(C and D Points)
•	Intersect	Intersect two segment that named AB and CD. (E point)
4	Distance or Length	Check the length of [AE] and [EB] line segment









PARALLEL LINE

CONSTRUCT A LINE PARALLEL TO THE GIVEN LINE GOING THROUGH POINT B

MENU	TOOL	PROCESS STEPS
		A
*	Line	ConnectA and C points with the "Line" tool.
\bigcirc	• Circle with Centre through Point	Draw a circle with the center it at A and cutting the line at two points.
•	Intersect	Define the points that formed with intersection of circle and lines (E and F points)
	Compasses	Drawa circle with radius equal to length AEand center B
•	Intersect	Define the points that formed with intersection of circle and line with the intersect tool (G point)
	Compasses	Construct a circle with radius equal to EFand center G
•	Intersect	Define the points that formed with intersection of circles that C and G- centered (H point)





• Line	Connect C and H points with the "Line" tool.
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PERPENDICULAR BISECTOR

CONSTRUCT A PERPENDICULAR BISECTOR TO THE GIVEN LINE SEGMENT

MENU	TOOL	PROCESS STEPS
- And	Segment	Draw [AB] line segment
\odot	• Circle with Centre through Point	Draw two circles with radius equal to length of AB and center it at A and B
•	Intersect	Define the points that formed with intersection of circles with the intersection tool (C and D points)
and a	Segment	Connecttwo points with the "Segment" tool (C and D points)
•	Intersect	Define the point that formed with intersection of segmentsAB and CD (E point). This must be midpoint of AB segment.
	Distance or Length	Check the length of AE and EB line segments
	🗸 Angle	Check the angles AEC or BEC









PERPENDICULAR LINE

DRAW A LINE (SEGMENT) THAT GOES THROUGH POINT A AND THAT IS PERPENDICULAR TO THE GIVEN LINE SEGMENT

MENU	TOOL	PROCESS STEPS
	A •	Draw a segment and mark arbitrary point A on it.
$\overline{\bullet}$	• Circle with Centre through Point	Draw a circle that with of any radius and with center it at A
•	Intersect	Define the points that formed with intersection of circle and line with the intersection tool (E and D points)
\bigcirc	• Circle with Centre through Point	Draw a circle that center it at D with radius is bigger than length of AE
\bigcirc	Compasses	Draw a circle that E centered radius is equal to the radius of the D centered circle
•	Intersect	Define the points that formed with intersection of circles that is centered it at E and D with the intersection tool (H and G points)
An Ar	✓ Segment	ConnectH and G points with the "Segment" tool.
	🔬 Angle	Check the angle DAH or HAE



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QUADRILATERAL

Form a quadrilateral to a rectangular

MENU	TOOL	PROCESS STEPS
	⊳ Polygon	Draw a randomquadrilateral ABCD
A	Midpoint or Centre	Draw the midpoint of the segments AB,BC,CD and DE(Points E,F,G and H)
×	🥕 Segment	Draw the segments EF,FG,GH and HE
×	🥕 Segment	Draw the segments AC and BD
▲ ●	Intersect	Find the intersection point I of the segments AC and BD
	Question1: What kind of shape is the EFGH ?	
	\land Angle	Draw the angles DIA and EHG

- Question2: If the EFGH rectangular is a rectangle what would be the shape of the initial uadrilateral.
- Question3: If the EFGH rectangular is a square what would be the shape of the initial quadrilateral.









DRAW A RECTANGLE WITH THE SAME AREA AS THE GIVEN TRIANGLE AREA

MENU	TOOL	PROCESS STEPS
	խ Polygon	Draw an arbitrary triangle ABC
+	Perpendicular Line	Construct perpendicular line to segment AB from vertex C
• ^A	Intersect	Mark the intersection of perpendicular line and segment AB – point D
•	Midpoint or Centre	Find the midpoint of segment CD – point E
+	Parallel Line	Construct the parallel line "g" through point E to segment AB
+	Perpendicular Line	Draw perpendicular lines from points A and B to paralel line "g"
• ^A	• ^A Point	Mark intersections of those lines – points F and G
	Polygon	Select vertices of rectangle A, B, G, F (then first vertex again)
	Area	Compare areas of rectangle ABGF and triangle ABC











CONSTRUCT A SQUARE WITH THE SAME AREA AS THE GIVEN RECTANGLE AREA

MENU	TOOL	PROCESS STEPS
		Construct an arbitrary rectangle "ABCD"
	🖍 Ray	Draw a rey on the segment "CD"
\bigcirc	• Circle with Centre through Point	Construct a circle with centre point "D" through point "A"
• ^A	Intersect	Define an intersect of the circle and ray "CD" – point "E"
• ^A	Midpoint or Centre	Define a midpoint of segment "EC" – point "F"
$\overline{\odot}$	Semicircle through 2 Points	Draw a semicircle with end points "E" and "C"
	✓ Ray	Draw a rey on the segment "AD"
• ^A	Intersect	Define an intersect of the ray AD and semicircle – point "G"





	📫 Regular Polygon	Select points "D" and "G" and draw regular polygon with 4 vertices – square "IDGH"
4	Area	Compare area of rectangle "ABCD" and square "IDGH"





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EUROGEBRA WORKSHEET

SQUARE

CONSTRUCT A SQUARE

MENU	TOOL	PROCESS STEPS
x	Line	Draw AB line
*	Perpendicular Line	Draw a perpendicular line thatgoes to A point to AB
•	• Circle with Centre and Radius	Draw a circle that radius equal to length of AB segment and centered at A
\odot	Intersect	Define the points that formed with intersections of circle and lines
\odot	Angle Bisector	Draw angle bisectors of EAD and DAF angles
•	\succ Intersect	Define the points that formed with intersections of circle and angle bisector lines
_**	✓ Segment	Combine the points that formed with intersected circle and angle bisectors
cm	Distance or Length	Check the length GA and GB line segments









TRIANGLE, LINE

Draw an equilateral triangle such that the segment ab is one of its sides

MENU	TOOL	PROCESS STEPS
- And	Segment	Draw AB line segment
\bigcirc	• Circle with Centre through Point	Draw two circles with radius equal to length AB and the centers must beA and B
•	Intersect	Define the points that formed with intersection of circles with the intersection tool. Give a name one of the intersections like C
	Polygon	Draw triangle by connecting A,B,C points
	\land Angle	Check the angles of the formed triangle



