EUROGEBRA WORKSHEETS

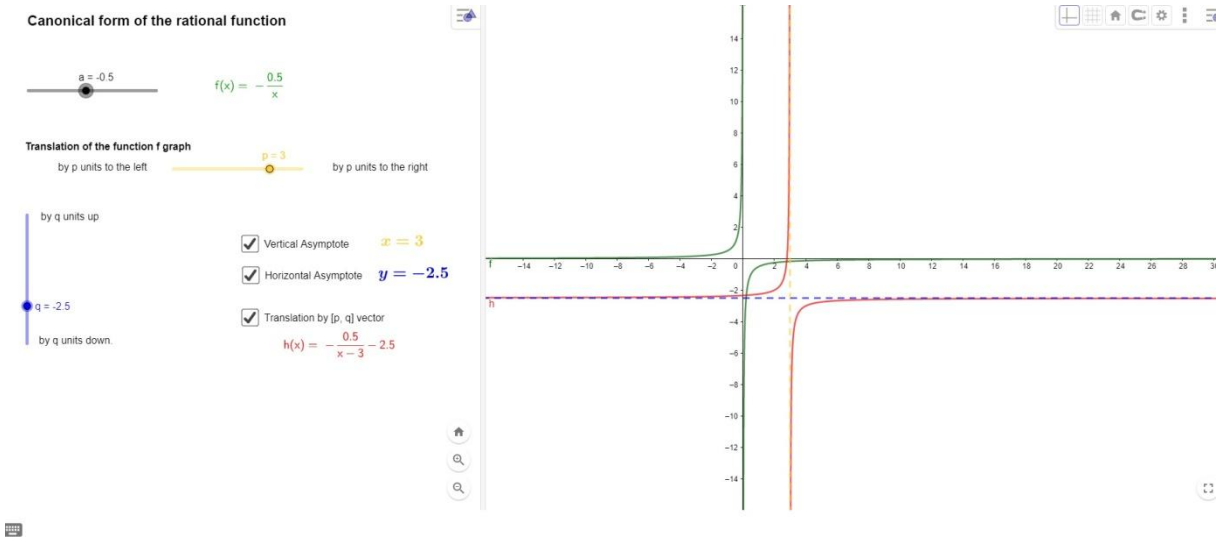
Analytical Geometry

EUROGEBRA WORKSHEETS

Introduction:

These worksheets were created within the Erasmus + project, Eurogebra.
Worksheets are in the field of mathematics and use the Geogebra program for individual mathematical tasks. The purpose is to use the program when teaching and explaining problems in mathematics and thus to approach the issue more clearly.
Worksheets are in the form of specific instructions and tools that will guide us to solve various tasks. In this way, students will get closer to a better understanding and modification of the given examples. Individual groups of worksheets can be combined with each other and create new subgroups according to the issues discussed. Some examples are followed by the solution of examples and free sheets for student notes.
Worksheets respect pedagogical documents related to the subject of mathematics. When working with worksheets, it is necessary to cooperate with teachers and coordinate their work.
In terms of content, we focused on geometric examples, where you can effectively solve problems and modify them in various ways. By formulating the tasks, we lead the students to understand the assigned tasks and to solve the tasks according to the individual steps in the worksheets.

Canonical form of the rational function – combo box



Instruction:

1. Turn the Algebra View off
2. Turn the Graphic View and Graphic View 2 on
3. Hide the axes in the Graphic View

ABC	Activate the Graphic View. Insert the static text that is a name of our construction, e.g. Canonic form of the rational function.
a = 2	Insert the slider a – values: -5 to 5, 0.1 increment
	Activate the Graphic View 2 and key y = a/x in the input bar. A graph of the function appears dependent on the factor a. Attn: you can always edit the object location – click the particular object, choose the Advanced tab, then Location (tick or uncheck the right View).
ABC	Insert the text: h(x) = f Choose f in the object list, activate the LaTex formula. Edit its features in the Advanced tab, key a≠0 as the Condition of the object displaying.
ABC	Insert the text: That is not a rational function. Edit its features in the Advanced tab, as the Condition of the object displaying key a=0 in, set it the same place as the text displaying the function formula.
a = 2	Insert the p slider, values: -6 to 6, 0.5 increment. Set the yellow colour. Match the slider location and the texts.

	Insert the text: Translation the graph of the function f.
	Insert the text: by p units to the left. Insert the text: by p units to the right. Set them adequately from left and right side of the slider p.
	Insert the vertical slider q (in the Slider tab choose the vertical one), values: -6 -6, 0.5 step. Change its colour blue and match the slider location and the texts.
	Insert the text: by q units up. Insert the text: by q units down. Set them adequately above and below the slider q.
	Activate the Graphic View 2, then key asymptote equation in the input bar. - vertical: x=p, set it yellow, style of the straight line – dashed - horizontal: y=q, set it blue, style of the straight line – dashed
	Activate the Graphic View 2, then key y = a/(x-p) + q in the input bar. The graph of the function appears that is translated by [p, q] vector. Change its colour red, set the style of the straight line 5.
	In the Graphic View insert the text: x= p, choose p in the object list. Change its colour yellow, tick the LaTex formula.
	In the Graphic View insert the text: y= q, choose q in the object list. Change its colour blue.
	Insert the combo box in the Graphic View. As the Caption insert the text: Vertical Asymptote, then from the list Choose object from the construction or choose from the list Indicate x=p
	Insert the combo box in the Graphic View. As the Caption insert the text: Horizontal Asymptote, then from the list Choose object from the construction or choose from the list Indicate y=q
	Insert the combo box in the Graphic View. As the Caption insert the text: Translation by [p, q] vector, then from the list Choose object from the construction or choose from the list Indicate y = a/(x-p) + q
	Polish the construction in terms of aesthetics, fix the objects.

Dispalying the sum of the triangle angles, its area and perimeter

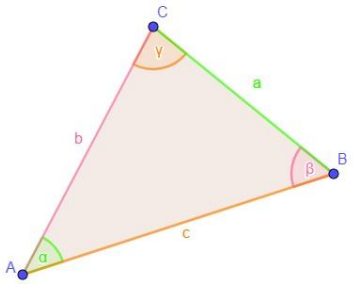
Instruction:

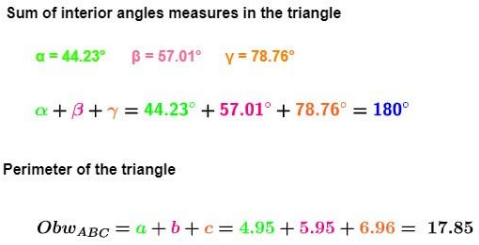
Open the Graphic View window

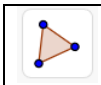
- in Options change Labelling to New Points Only. Type size: 16.

- Hide the Axes and Grid.

The expected effect of the construction:

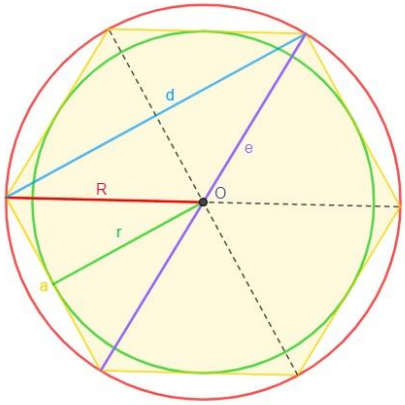




	Draw a triangle ABC.
	Show the side labels of triangle (after choosing the tool, click: triangle sides) or click RMB on the particular side and choose the option: Show a label.
	Draw interior angles of triangle. Edit their features, click RMB and in the Basic tab tick an option: angle between 0° and 180°
	Turn the Graphic View 2 on.
	In the Graphic View 2, insert the text 1: Sum of interior angles measures in the triangle.
	In the Graphic View 2, insert the next three dynamic texts that will display measures of the particular angles.

	Choose Greek lettering from the Symbol tab, whereas the angle from the Object  tab. Change the colour of the particular text corresponding to its angle, e.g. α – olive, β – sky-blue, γ – red.
	Activate the Input bar.
	Key: α + β + γ in the input bar. Instructions: symbols of Greek alphabet are to the right of the input bar. In the Algebra View there will be number δ equal to the sum of interior angles measures in the triangle.
	You can key the extract of the text in different colours. Tick the LaTex option and insert the following formula: \textcolour{colour name}{the right extract of the text}
	Insert the colourful dynamic text in the Graphic View 2.
	In the Graphic View 2, insert the text: Perimeter of the triangle.
	Insert the colourful dynamic text in the Graphic View 2.
	In the Graphic View 2, insert the text: Area of the triangle = polygon1 Choose the polygon1 from the Object window.

Regular Hexagon



☑ Regular hexagon with the side a

each regular hexagon can be divided into 6 equilateral triangles with the side length a

☑ : A circle circumscribed about a regular hexagon with the side a

R = a

☑ An inscribed circle of a regular hexagon with the side a

$$r = \frac{a\sqrt{3}}{2}$$

☑ A shorter diagonal

$$d = 2r = 2 \cdot \frac{a\sqrt{3}}{2} = a\sqrt{3}$$

☑ A longer diagonal

e = 2R = 2a

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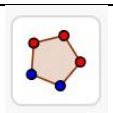
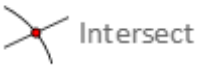
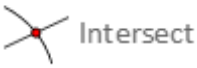
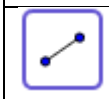
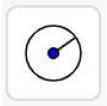
Initial activities:

- Menu – Options – Labelling tick: All new objects

Instruction:

	Draw a regular hexagon. After ticking two vertices of the polygon, e.g. A, B in the query window key the number of sides that is 6. Edit its features. In the Style tab set Thickness of the straight line 5, in the Colour tab set it yellow and set the Transparency 0.
	Draw a circle going through the points A, B, C. Edit its features. In the Style tab set Thickness of the straight line 5, in the Colour tab set it red.
	Draw diagonals of the hexagon that is AD, BE and CF segments. Edit its features, change the Style of the line – dashed.
	Define the intersection point G, change its name into O.
	Draw the segment AO. It is a circumradius. Edit its features in the Basic tab: as Caption key R, choose as Show a Label: Caption. In the Style tab, set Thickness of the straight line 5, in the Colour tab set it red.
	Draw a line p that is perpendicular to AB segment and crossing the point O.

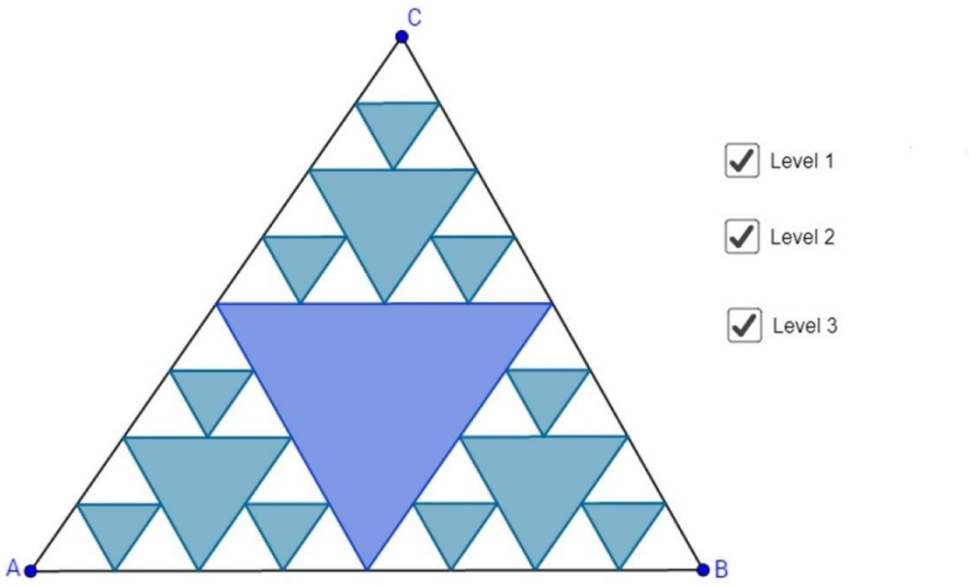
	Define the point G that formed with intersection of line p and side AB.
	Draw the segment OG, it is an inradius. Edit its feautures, in the Basic tab: as Caption key r, choose as Show a Label: Caption. In the Style tab, set Thickness of the straight line 5, in the Colour tab set it green.
	Draw an inscribed circle of a hexagon that is a circle with the center point O and the radius r. Edit its features, in the Style tab set Thickness of the straight line 5, in the Colour tab set it green.
	Draw a segment that is a shorter diagonal of the hexagon. Edit its features, in the Style tab set Thickness of the straight line 5, in the Colour tab set it blue, as Caption key d , tick the Show a Caption option in the Basic tab.
	Draw a segment that is a longer diagonal of the hexagon. It will cover with one of the earlier drawn segments. Edit its features, in the Style tab set Thickness of the straight line 5, in the Colour tab set it purple, as Caption key e , tick the Show a Caption option in the Basic tab.
	Draw a segment AB that is a side of the hexagon.
	Hide the straight line p.
	Hide all unnecessary labels and objects.

Insetting of the combo box

	Insert the text: each regular hexagon can be divided into 6 equilateral triangles with the side length a.
	Insert a combo box Show/ Hide an object with Caption: Regular hexagon with the side a. Tick in the construction or choose objects from the list: Hexagon, its sides and its longer diagonals and their intersection point.
	Insert the text: R = a. Set its colour red, choose the embolden option.
	Insert a combo box Show/ Hide an object with Caption: A circle circumscribed about a regular hexagon with the side a. Tick in the construction or choose objects from the list: Circle C, segment R and previously added text.
	Insert the text using the LaTex option. The green colour, embolded.

	Insert a combo box Show/ Hide an object with Caption: An inscribed circle of a regular hexagon with the side a. Define objects: a corresponding circle, its radius r and previously added text.
	Insert the text using the LaTex option. The blue colour, embolded.
	Insert a combo box Show/ Hide an object with Caption: A shorter diagonal, Marked objects: A shorter diagonal and previously added text.
	Insert the text: e = 2R = 2a The purple colour, embolded.
	Insert a combo box Show/ Hide an object with Caption: A longer diagonal, Marked objects: A longer diagonal and previously added text.
	Polish the construction in terms of aesthetics.

Sierpinski Triangle

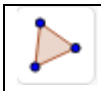
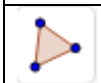


Initial activities:

- Hide the Grid and Axes of the coordinate system

- Set Labelling – New Points Only

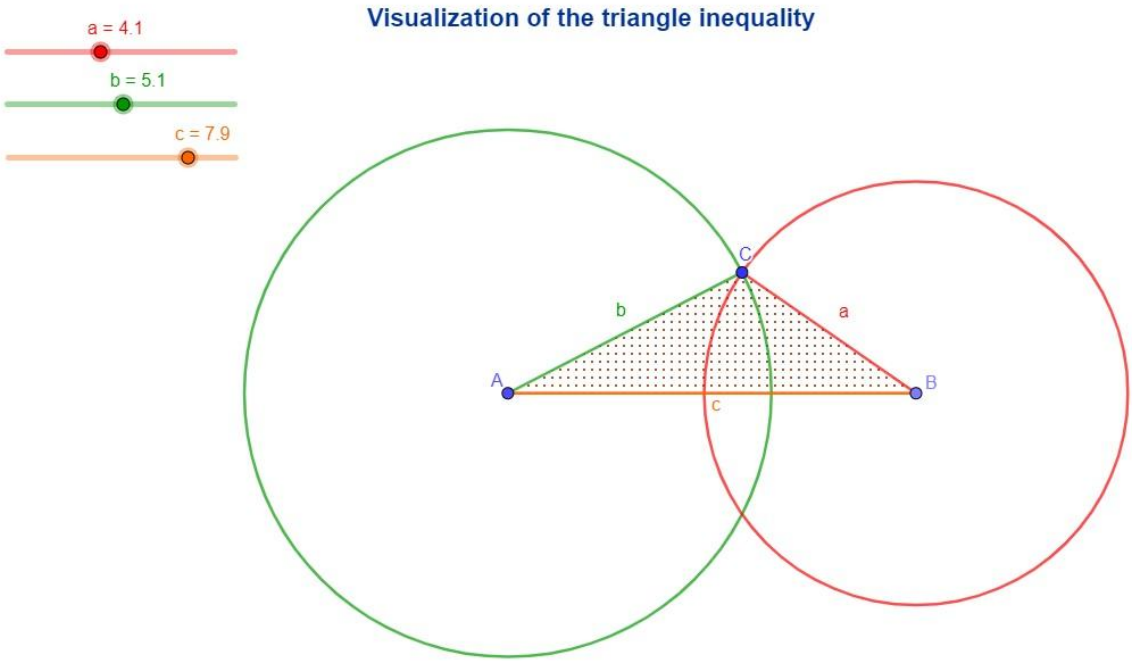
Instruction:

	Construct a triangle ABC. Edit its features. In the Colour tab set the black one, decrease Transparency to 0.
	Define the point D – the centre of triangle side AB Define the point E – the centre of triangle side BC Define the point F – the centre of triangle side AC
	Draw a triangle DEF. Edit its features. In the Colour tab set the blue one, decrease Transparency to 50%.
	Create a new tool named Sierpinski. Output objects: points D, E, F, triangle DEF, sides of triangle DEF. Input objects: pints A, B, C. Name: Sierpinski Instructions: Click three noncollinear points.
	Use the tool to three blue triangles: ADF, DBE and FEC to create the second level of Sierpinski Triangle.

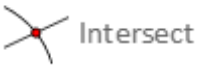
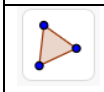
	Use the tool to previously created triangles to form the third level of Sierpinski Triangle.
	Hide all the points except for A, B, C.
	Insert the combo box Show/Hide Object with Caption – Level 1. Choose appropriate objects from the triangle construction and its sides. Insert the next two combo boxes likewise – Level 2 and Level 3.

Visualization of the triangle inequality

The expected effect of the activities below:

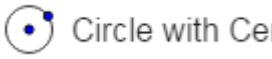


	Hide the Algebra View, Axes and Grid in the Graphic View, Labelling – new points only.
	Display the Navigation bar of steps at the bottom of the Graphic View. Choose the Navigation bar from the context menu in the Graphic View (RMB in the Graphic View window).
ABC	Insert the text: Triangle construction from three segments.
a = 2	Insert sliders a, b, c corresponding to the length of triangle sides. Slider range: 0…10, 0.1 step.
a	Insert the segment with the starting point A and length c.
	Insert circles: with the center point A and radius b, and with the center point B and radius a.

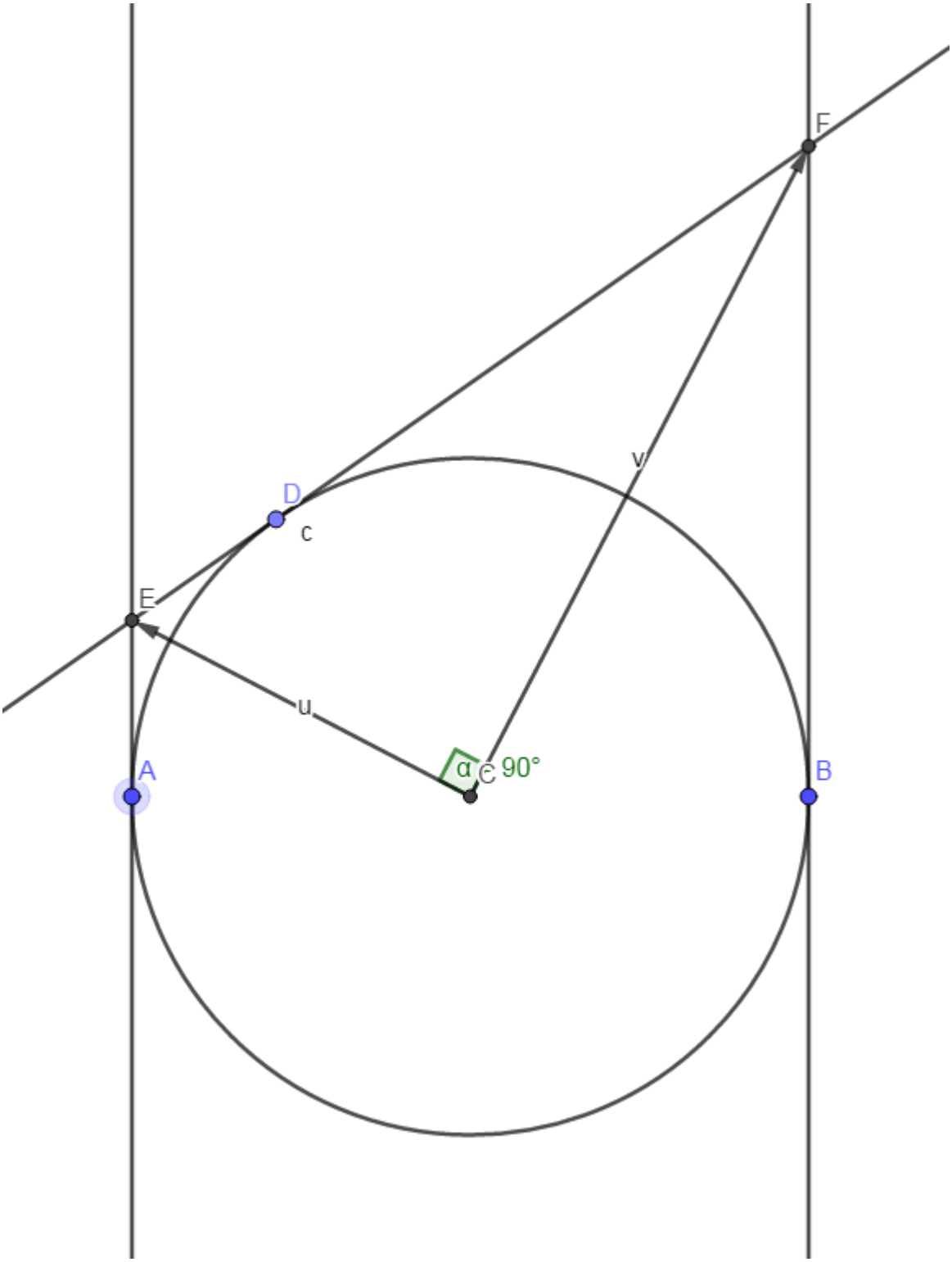
	Define the point C – intersection of the circles.
	Draw a triangle ABC.
	Match the colours corresponding to particular objects, e.g. slider a, segment a and circle with radius a – red; slider b, segment b and circle with radius b – blue; slider c and segment with the length of c – green.
	Turn labels of triangle sides on as their Captions. (Two objects cannot have the same name – if the slider is named a, the segment can only have the label named a – not the name a).
	Using the Navigation bar at the bottom of the window, play the construction step by step. Try the AutoPlay.

EUROGEBRA WORKSHEET

CIRCLE THE TANGENTS AND VERTICAL VECTORS

MENU	TOOL	PROCESS STEPS
	 Point	**Click on the board to see the A point**
	 Point	**Click on the board to see the B point**
	 Midpoint or Centre	**Click on the A and B points to find the midpoint C of the AB segment**
	 Circle with Centre through Point	**Select the C point and then the A point to draw the C centre circle.** The circle passes through the antidiametric points A and B
	 Tangents	**Select the A point and then the C circle to draw the f tangent of the circle**
	 Tangents	**Select the B point and then the C circle to draw the g tangent of the circle**
	 Point	**Click on the circle to draw another point D, different from the A and B points**
	 Tangents	**Select the D point and then the C circle to draw the h tangent of the circle**

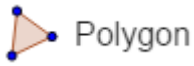
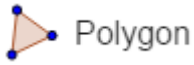
	 Intersect	**Click on the f and h tangents to find the intersect point E**
	 Intersect	**Click on the g and h tangents to find the intersect point F**
	 Vector	**Click on the C and E points to draw the u vector**
	 Vector	**Click on the C and F points to draw the v vector**
	 Angle	**Click on F, C and then E points to measure the FCE angle**
Write in the input cell "u·v" and then click "enter" to see that u·v = a = 0 and notice that the product uv is constant equal to 0		
1 st task : why are the tangents f, g parallel ?		
2nd task : the product uv = 0 constantly , why ?		
3rd task : why are the u, v vectors vertical ?		

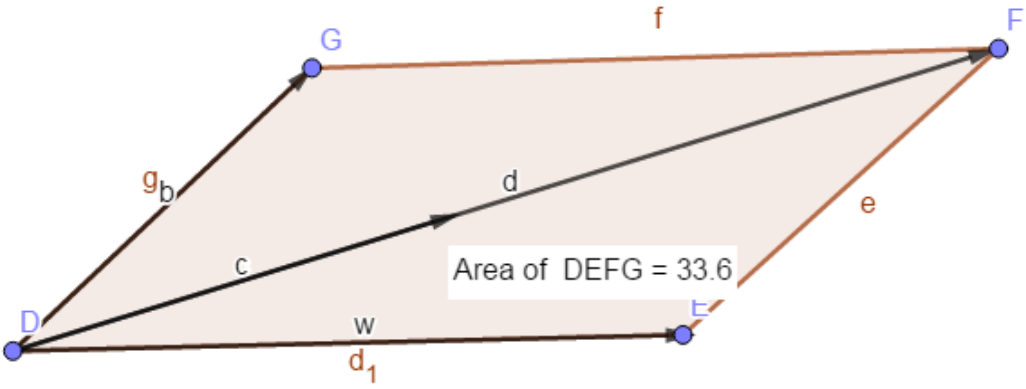

F
v
D
c
E
u
A
α
C
90°
B

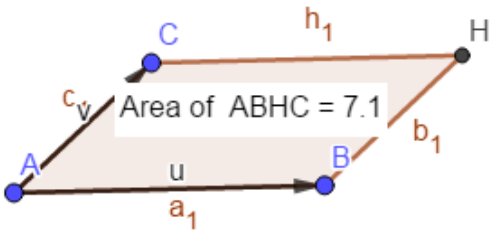
EUROGEBRA WORKSHEET

TITLE : COLLINEAR VECTORS AND RECTANGLE

MENU	TOOL	PROCESS STEPS
		Click on the board to see the A point
		Click on the board to see the B point
		Do the same , to see the C point
		Select the A point and then the B point to draw the u vector
		Select the A point and then the C point to draw the v vector
		Click on the board to see the a slider, min = -5 and max = 5
Write in the „input“ cell „w = a·u“ and the w vector will appear . w // u (collinear vectors)		
Write in the „input“ cell „b = a·v“ and the b vector will appear . b // v (collinear vectors)		
Write in the „input“ cell „c = u + v“ and the c vector will appear . c is the vector sum of the u, v		

Do the same dy writing „d = w + b“ The c and d vector have the same start point D		
1st task : Explain why the „c“ and „d“ vectors are collinear		
	 Point	Click successively the end of w, d and b vectors to draw the E, F, G points
	 Polygon	Click on D, E, F and G points to draw the DEFG rectangle
	 Parallel Line	Click on AC side and the B point to see the parallel line h //v
	 Parallel Line	Click on AB side and the C point to see the parallel line i //u
	 Intersect	Click the h and then i lines to draw the intersection point H
	 Polygon	Click on A, B, H and C points to draw the ABHC rectangle
	 Area	Click on the DEFG rectangle to messure its area (q1). Repeate the procedure to messure the area of the ABHC rectangle (q2).
Calculate in the „input“ cell „j = q2/q1“ and then „ $k = 1/a^2$ „ . Note that : „ j = k „		
2nd task : Explain why the two rectangle are similar and 3rd task : why the „ j“ and „ k“ rates are equal		


f
G
F
g
b
d
e
c
Area of DEFG = 33.6
D
w
d1


h1
C
H
c
v
Area of ABHC = 7.1
b1
A
u
B
a1


a = 2.2

EUROGEBRA WORKSHEET

DIFFERENTIATION OF SIN AND COS FUNCTIONS

MENU	TOOL	PROCESS STEPS
		From the algebra view type in f(x)= sin x
		Type in f'(x).
		From the algebra view type in g(x)= cos x
		Type in g'(x).
		From the algebra view type in h(x)= -cos x Type in h'(x)
		From the algebra view type in j(x)= -sinx Type in j'(x)
		From the algebra view type in f(x)= sin 2x

Questions:

1.From the algebra view type in f(x)= sin x. The x scale is in radians

2. Type in f'(x). What do you notice? The derivative is

3. From the algebra view type in g(x)= cos x. The x scale is in radians

4. Type in g'(x). What do you notice? The derivative is

5. From the algebra view type in h(x)= -cos x. The x scale is in radians

6. Type in h'(x). What do you notice? The derivative is

7. From the algebra view type in f(x)= -sinx. The x scale is in radians

8. Type in f'(x). What do you notice? The derivative is -

9. Repeat steps 1-8 but instead use 2x in place of x. What do you notice? Can you generalise your solution for differentiating cos nx and the sin nx where n is a positive integer.

10 . What about if n was a negative integer?

11. How would you integrate the functions in steps 1-10?

$f(x) = \sin(x)$

$f'(x) = f'(x)$
$\rightarrow \cos(x)$

$g(x) = \cos(x)$

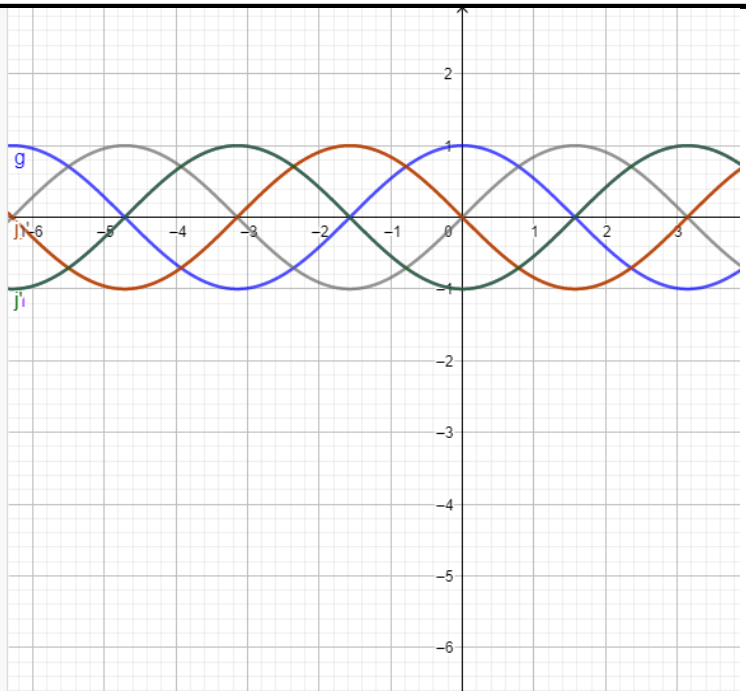
$g'(x) = g'(x)$
$\rightarrow -\sin(x)$

$h(x) = -\cos(x)$

$h'(x) = h'(x)$
$\rightarrow \sin(x)$

$j(x) = -\sin(x)$

$j'(x) = j'(x)$
$\rightarrow -\cos(x)$



EUROGEBRA WORKSHEET

DIFFERENTIATION OF IN X AND EXP FUNCTIONS

MENU	TOOL	PROCESS STEPS
		From the algebra view type in f(x)=log(e,x)
		Type in f'(x).
		From the algebra view type in g(x)= e^x
		Type in g'(x).
		From the algebra view type in h(x)= log(e,2x) Type in h'(x)
		From the algebra view type in j(x)= e^2x Type in j'(x)

Questions:

1.From the algebra view type in f(x)= log(e,x)

2. Type in f‘(x). What do you notice? The derivative is

3. From the algebra view type in g(x)= e^x.

4. Type in g‘(x). What do you notice? The derivative is

5. From the algebra view type in h(x)= log(e,2x)

6. Type in h‘(x). What do you notice? The derivative is

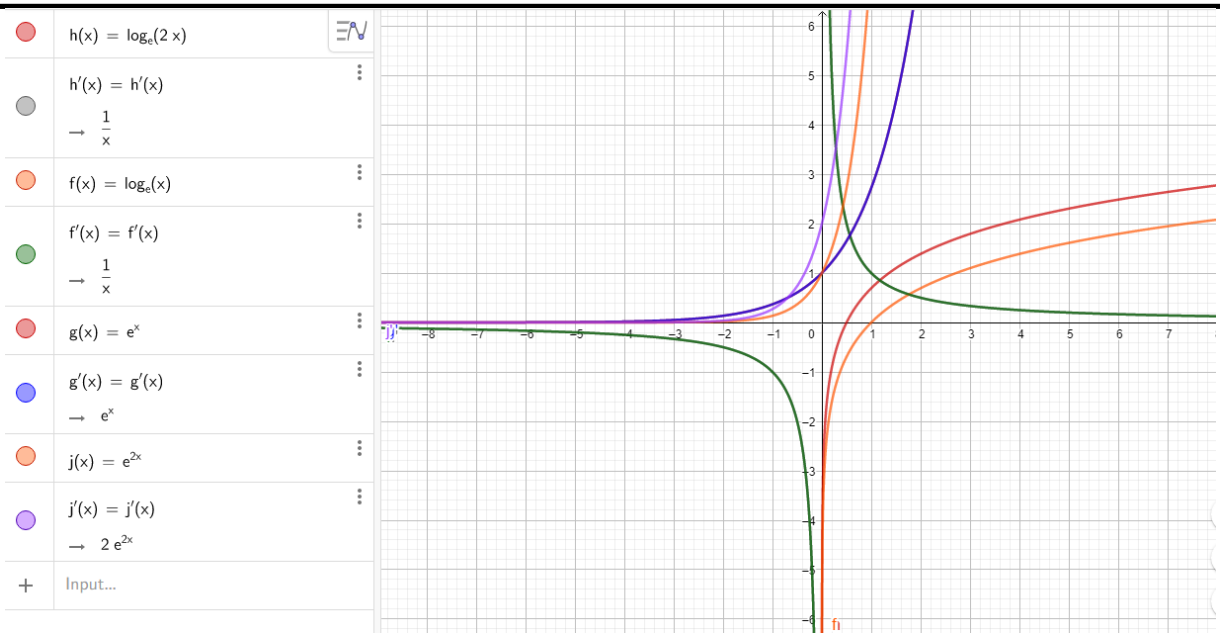
7. From the algebra view type in j(x)= e^2x

8. Type in j‘(x). What do you notice? The derivative is -

9. Can you generalise your solution for differentiating ln nx and the e^ nx where n is a positive integer.

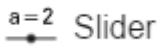
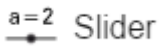
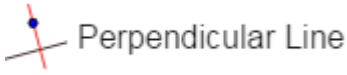
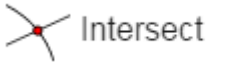
10 . What about if n was a negative integer?

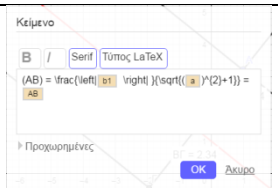
11. How would you integrate the functions in steps 1-10?

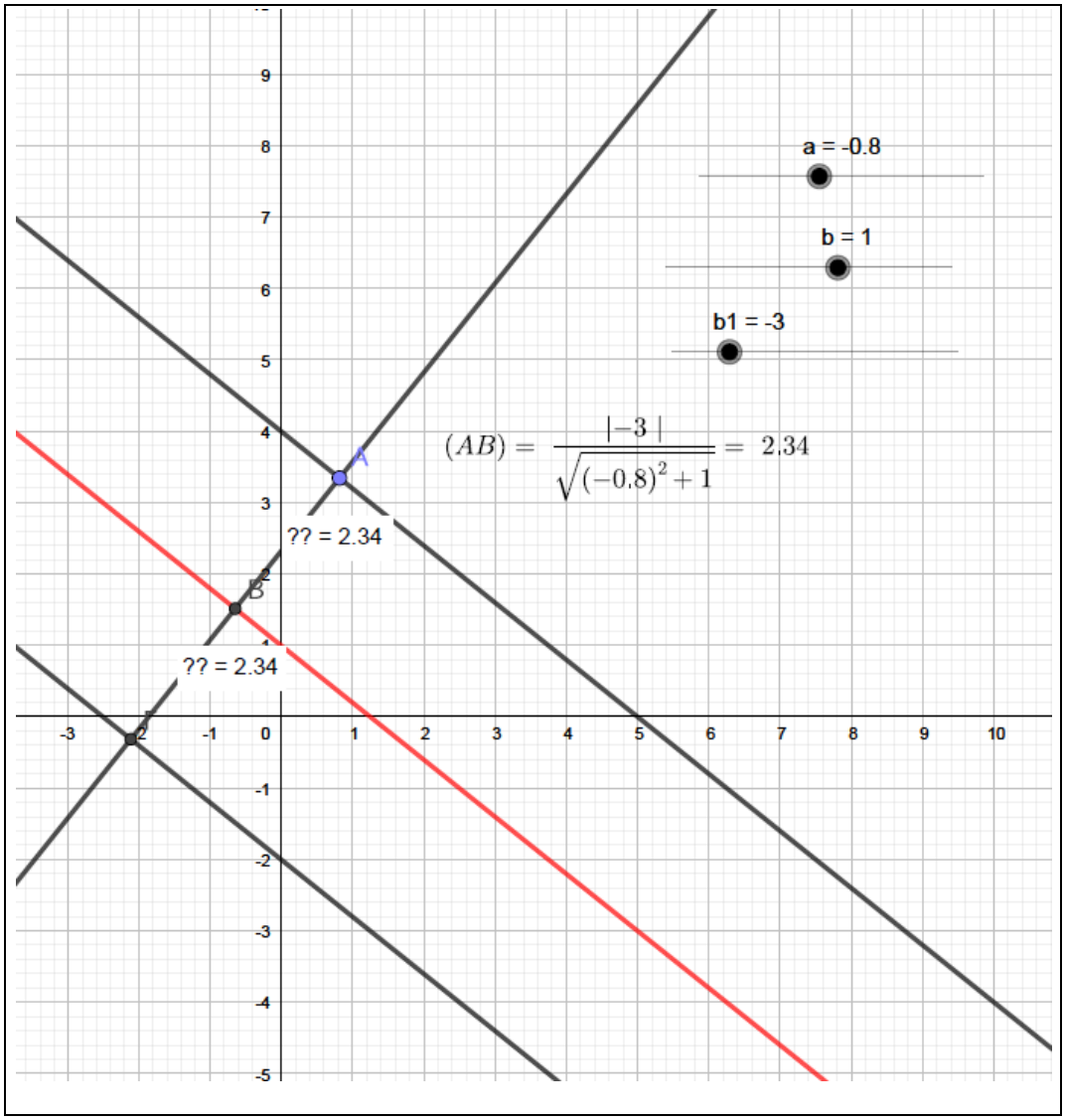

h(x) = log_e(2 x)
h'(x) = h'(x)
→ 1/x
f(x) = log_e(x)
f'(x) = f'(x)
→ 1/x
g(x) = e^x
g'(x) = g'(x)
→ e^x
j(x) = e^(2x)
j'(x) = j'(x)
→ 2 e^(2x)
Input...

EUROGEBRA WORKSHEET

TITLE : INTERMEDIATE LINE AND DISTANCES

MENU	TOOL	PROCESS STEPS
		**Select an „a“ slider, min = - 5, max = 5**
		**Select a „b“ slider, min = - 5, max = 5**
		**Select a „b1“ slider, min = - 5, max = 5**
Write in the „input“ cell the equation : y = a·x + b to draw a line f		
Write in the „input“ cell the equation : y = a·x + b + Ib1I to draw a parallel line g // f		
Write in the „input“ cell the equation : y = a·x + b - Ib1I to draw the second parallel line h // f		
		**Select an A point on line g**
		**Click on the A point and f line to draw the perpedicular line i**
		**Click on f and i lines to find the intersection point B**

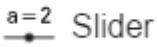
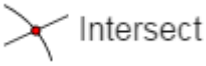
	 Intersect	**Click on h and i lines to find the intersection point C**
	 Distance or Length	**Click on A and B points to messure the (AB) distance of the two lines g and f**
	 Distance or Length	**Click on C and B points to messure the (BC) distance of the two lines h and f**
Note that the f line is the Intermediate line of the lines g and h		
Calculate in the „input“ cell the distance AB = $\frac{\|b1\|}{\sqrt{a^2+1}}$		
	ABC Text	 **Write „(AB) =** $\frac{\|b1\|}{\sqrt{a^2+1}} = AB$“
1st task : explain why the f, g, h lines are parallel		
2nd task : explain why the f line is the intermediate line of the g, h lines		
3rd task : prove the equality AB = $\frac{\|b1\|}{\sqrt{a^2+1}}$		


a = -0.8
b = 1
b1 = -3
$(AB) = \frac{|-3|}{\sqrt{(-0.8)^2+1}} = 2.34$
A
?? = 2.34
B
?? = 2.34

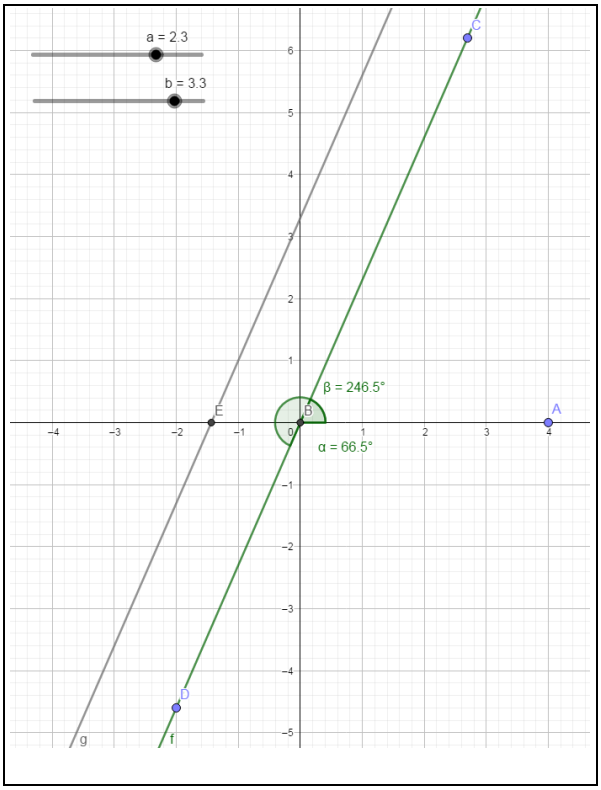
EUROGEBRA WORKSHEET

LINEAR EQUATIONS

MENU	TOOL	PROCESS STEPS
	 Slider	**Cilck on the geogebra board to define a slider „a“ , set min = - 5 and max = 5.**
Write in the input cell the function „ a·x ” to create a line f. The coefficient “a” of the fuction is the slope of the line you drew.		
Left click on the slider´s dot and mone it, to change the slope of the line. Set the slider a = 2		
	 Point	**Select the A point on the Ox axis, then B point on (0, 0) and at last the C point on the line, above x´x axis.**
	 Angle	**Click on the A, B and C points to measure the angle formed by the line with the axis x´x.** Notice that the “a” angle is acute.
Left click on the slider and move the dot to change the slope of the line. Set the slider a = - 2		
	 Point	**Select the D point on the line, above x´x axis.**
	 Angle	**Click on the A, B and D points to measure the angle formed by the line with the axis x´x.** Notice that the “b” angle is blunt.
1st task : What is the solution of the equation ax = 0 , a ≠ 0. (show the solution in the graph)		

		**Cilck on the geogebra board to define a slider „b“ , set min = - 5 and max = 5.**
Write in the input cell the function „ a·x + b ” to create a line g. The coefficient “a” of the two lines is the same. Notice that the two lines are parallel, if b ≠ 0. Notice that the two lines are identical, if b ≠ 0.		
		**Click on the g line and the x′x axis to see the intersection point E**

2nd task : What is the solution of the equation ax + b = 0 , a ≠ 0.
(show the solution in the graph)



5th mobility – on line
1st day Thu 22/06/2021

EUROGEBRA WORKSHEET

TITLE : linear programming (linear inequalities system)

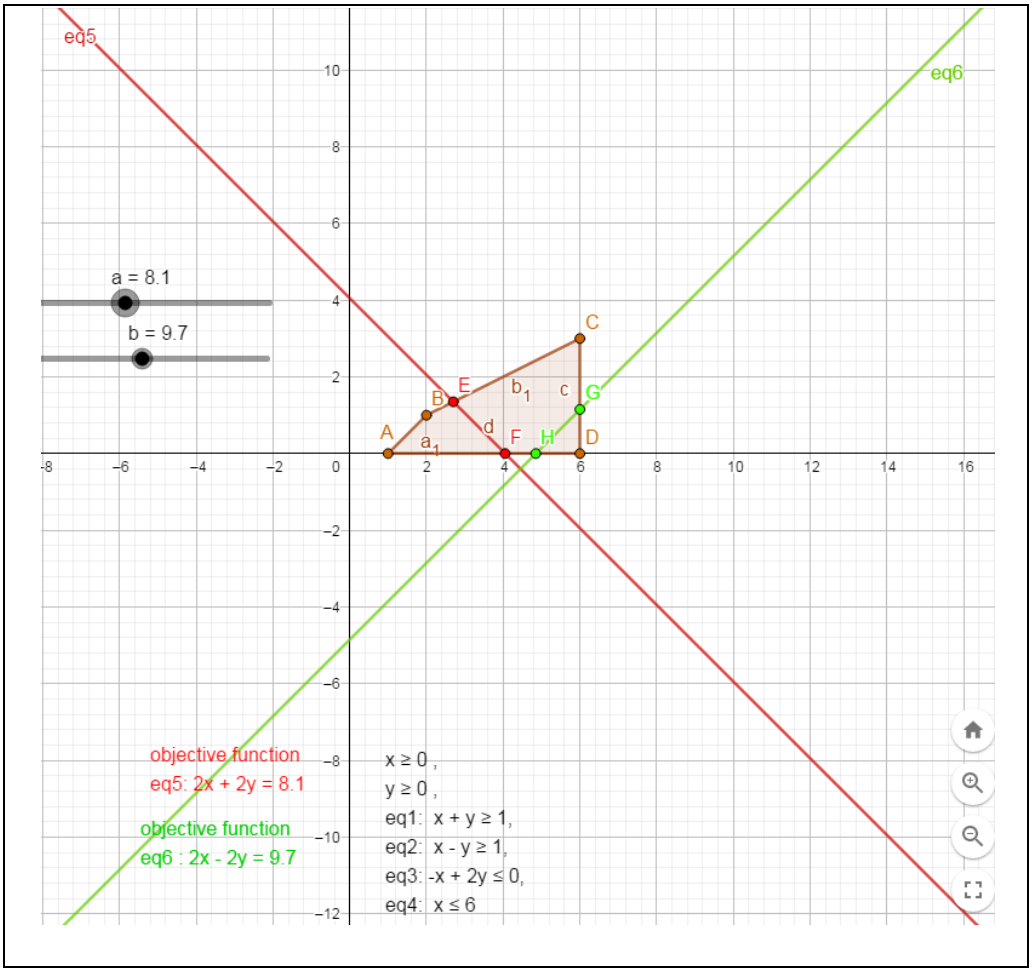
MENU	TOOL	PROCESS STEPS
	Slider	Cilck on the geogebra board to define a slider „a“ and „b“ , set min = 0 and max = 20.
Write in the input cell „ x + y = 1 ” (eq:1), “x – y = 1” (eq:2), “ – x + 2y = 0” (eq:3) and finally “x = 6” (eq:4)		
	 Intersect	A point, is the intersection of xAxis and eq2, B point, is the intersection of eq:3 and eq2, C point, is the intersection of eq:3 and eq4 and D point, is the intersection of xAxis and eq4
	 Polygon	By clicking A, B, C and D points , create the convex polygon ABCD. The polygon is the solution of the inequality system $x \geq 0$, $y \geq 0$, eq1: $x + y \geq 1$, eq2: $x - y \geq 1$, eq3: $-x + 2y \leq 0$, eq4: $x \leq 6$
Write in the input cell „ 2x + 2y = a ” (eq:5), (the objective function).		
	 Intersect	Click inside the ABCD polygon and eq:5, to see the intersection points E anf F.

1st task : By moving "a" slider can you find the minimum and the maximum of the objective function eq:5 ?
[remember that the eq:5 must intersect the polygon]

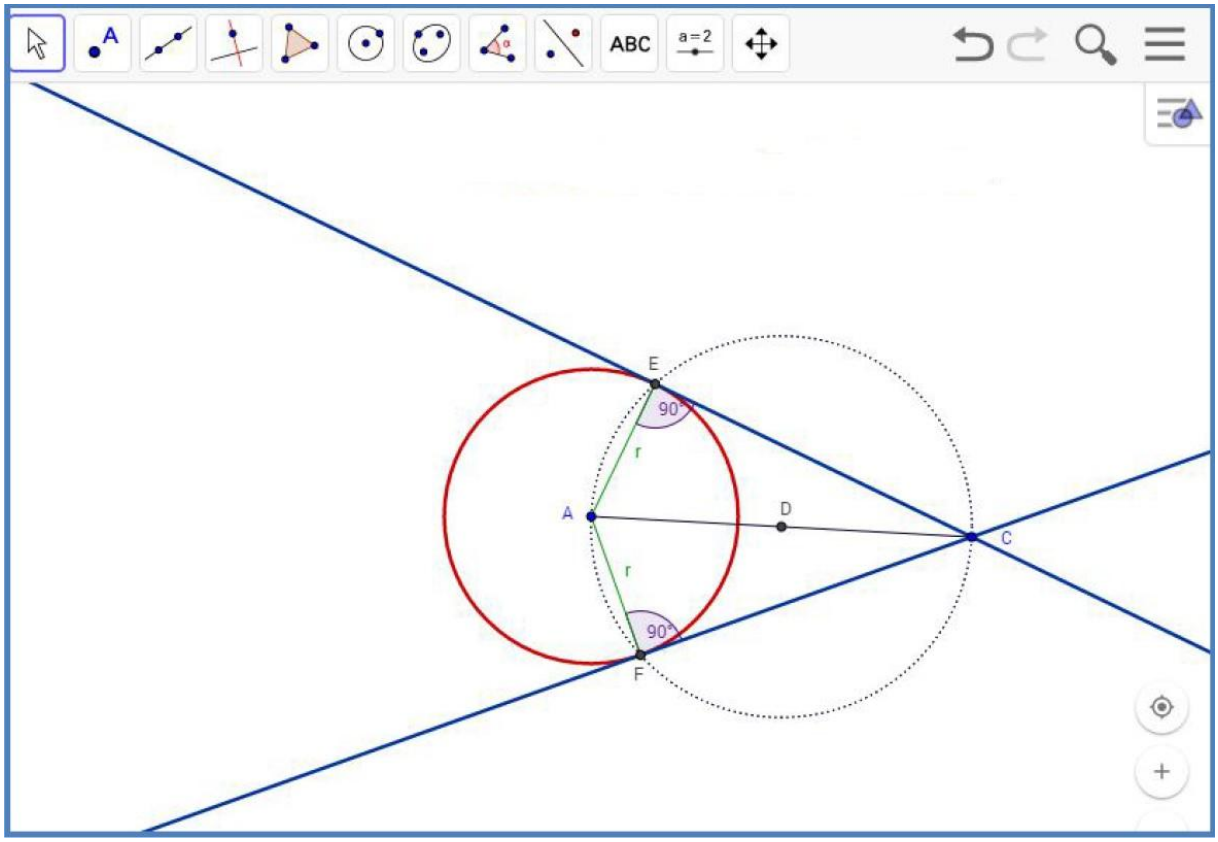
Write in the input cell another objective function „ 2x - 2y = b " (eq:6).

A	Intersect	**Click inside the ABCD polygon and eq:6, to see the intersection points G anf H.**

2nd task : By moving "b" slider can you find the minimum and the maximum of the objective function eq:6 ?
[remember that the eq:6 must intersect the polygon]

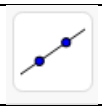


Tangents to a circle construction



- Open a new GeoGebra file
- Hide axes and grid
- Change the setting of the Objects label (choose Labelling from the menu Options, and then Only New Points).
- Follow the steps:

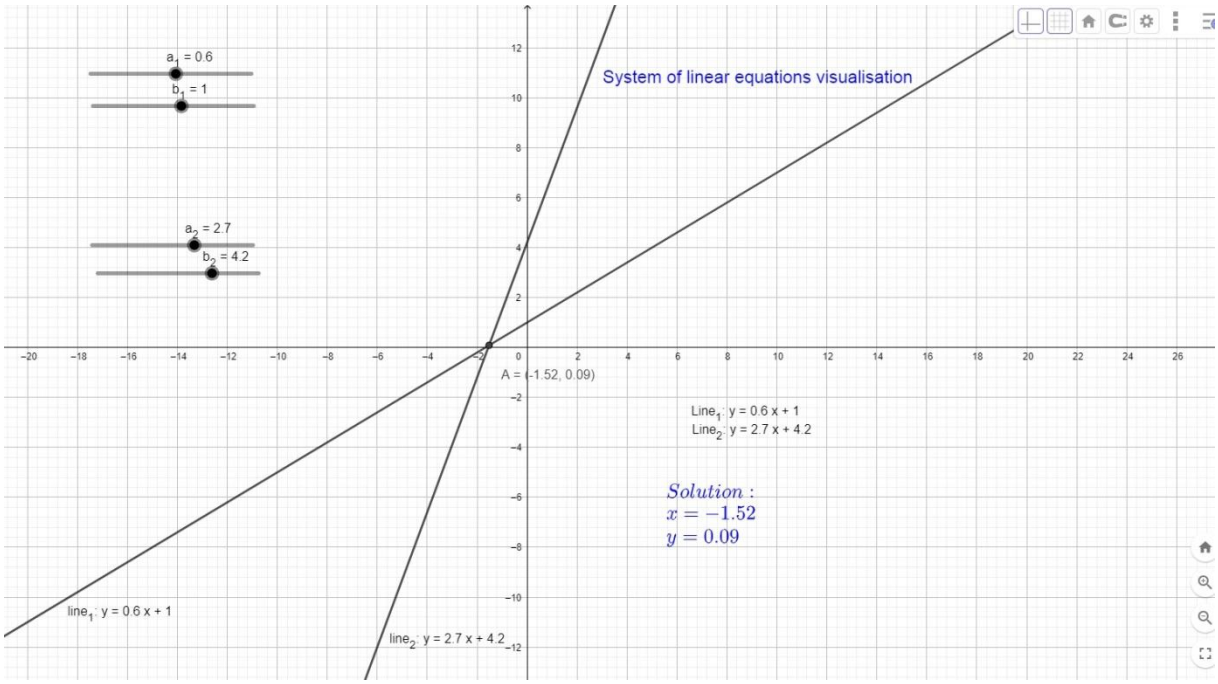
No	TOOL	PROCESS STEPS
1.		Choose the tool *Circle with centre through point*, draw a circle with A center passing through B point.
2.		Draw C point which is located outside the circle
3.		Using the tool *Segment* match the circle center A with a point C, segment *a* was created
4.		Find the center of segment *a*. Use the tool *Midpoint* or *Centre,* you will get point D.
5.		Construct a circle with centre at point D and passing through point C.
6.		Mark the intersection of both circles, points E and F were created.

7.		Using the tool *Line* draw two tangents to a circle: EC and FC
8.		Format the object
9.		Using the tool *Move* check if the construction was made correctly.

Additional activities

10.		To confirm that the lines are tangents to a circle, use the tool *Segment* and define the radius of the circle (join points A with E and A with F). Use *Caption* to have label **r** on both segments.
11.		Mark the angle between a tangent and a radius by clicking on verticles one by one clockwise (then marking will appear inside the angle)

System of linear equations visualisation



Create the applet, which will illustrate the system of linear equations visualisation.

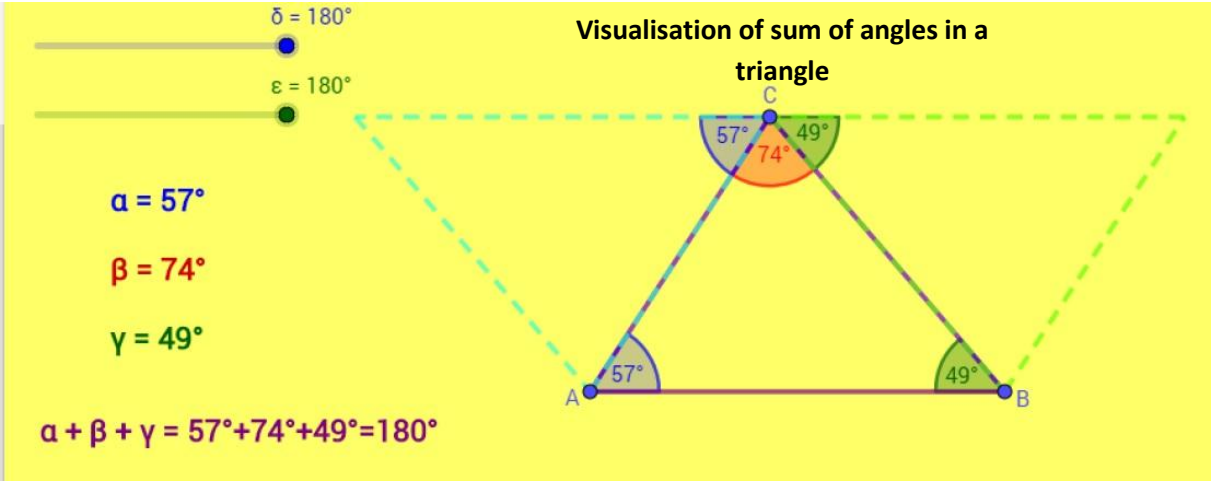
No	TOOL	PROCESS STEPS
1.	ABC	Insert text 1: *System of linear equations visualisation*
2.	a=2	Insert slider for **a_1** (a_1), interval between -10 and 10, increment 0.1.
3.	a=2	Insert slider for **b_1** number (b_1); interval between -10 and 10; increment 0.1.
4.		Show Input Bar (Menu – View – Input Bar). In the Input Bar (in the bottom under the graphics window) type in linear equation of **$line_1$:** *line_1: y=a_1x+b_1* line_1: y=a_1x+b_1
5.	a=2	Insert slider for **a_2** (a_2), interval between -10 and 10, increment 0.1.
6.	a=2	Insert slider for **b_2** number (b_2); interval between -10 and 10; increment 0.1.
7.		In the Input Bar type in linear equation of **$line_2$**

		line_2: y=a_2x+b_2. Click the right mouse button on $line_1$ and choose **Settings –** card **Basic – Show label: Name and Value (look point 4).**
8.		Insert dynamic texts showing the formula of Line 1 and Line2. 1. **Text 2**: Line_1: *line_1* 2. **Text 3** Line_2: *line_2* Attention!!! *line_1* an *line_2* choose from the scroll list Advanced
9.		Define the intersection of $line_1$ with $line_2$, using the tool *Intersect* and clicking on the first and then the second line. Point A will appear in the intersection (Show the label Name and Value).
10		Insert dynamic text 4: Solution: x=*x(A)* (x(A) defines coordinate x of point A) y=*y(A)* (y(A) defines coordinate y of point A) **ATTENTION!!!** In case of texts *x(A)* and *y(A)* use *empty formula box* from the scroll list *Advanced.* Type in everything in one line and in the end seperate the texts using Enter.
11.		Format the texts. Here are a few ways of formatting objects: 1) Click on the object you want to format with a left mouse button, a shortcut bar which you can use to format will appear. 2) Click on the object with a right mouse button, choosing *Settings* and appropriate tab from the context menu. The text can be bold, its size and font can be changed.

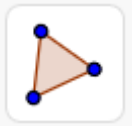
Tips:

- Show the label Name and Value for $line_1$ and $line_2$.
- Change the colours of $line_1$ and $line_2$.
- Adjust text colour to $line_1$ and $line_2$.
- After placing texts, place it choosing right mouse button and clicking on the text Fix Object.
- If you use LaTeX Formula and you want to insert Enter in the text use: \\, whereas for Space use: \.
- Scroll the slider observing how the solution of coordinate system and its lines are changing

Sum of angles in the triangle visualisation

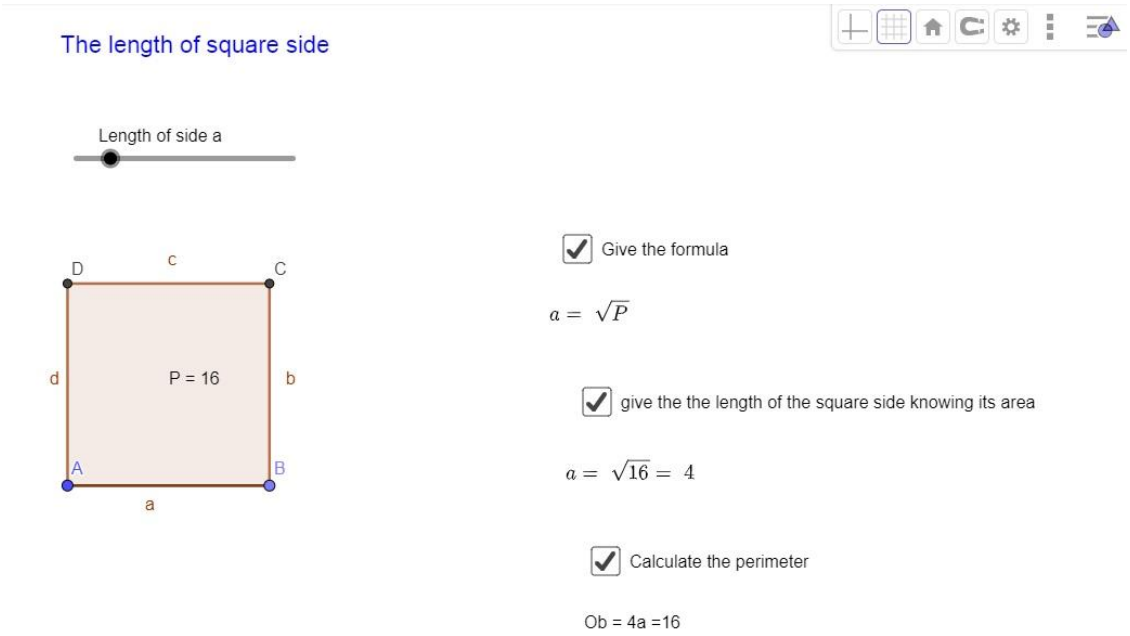


- Set the decimal place to 0 (Menu – Options – Rounding)

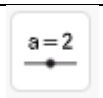
No	TOOL	PROCESS STEPS
1.	ABC	Insert text: Sum of angles in the triangle
2.		Draw a triangle ABC, using the tool *Polygon*
3.		Define internal angles of the triangle α, β,γ, using the tool *Angle* and choosing appropriate vertices BAC, ACB, CBA. Show the value of the angles.
4.		Create a slider for angle δ MIN 0°MAX 180° increment 10°
5.		Create a slider for angle **ε** MIN 0°MAX 180° increment 10°
6.		Using the tool *Midpoint*, find the midpoint D of segment AC and the midpoint E of segment CB.
7.		Using the tool *Rotate around point*, rotate the triangle choosing ABC triangle (clicking in the center of the triangle), then midpoint D and give δ angle [counterclockwise]
8.		Rotate the triangle ABC around E point by ε angle [counterclockwise]
9.		Set the δ and ε angle sliders to (for example) 140
10		Create ζ angle choosing points B'A'C' and η angle choosing C'_1,B'_1,A'_1 Choose Angle settings: Show label – Value.
11.		Hide all the points except A, B, C opening Algebra View and

		clicking on particular object.
12.		Turn off concave angle by clicking on the object with the right mouse button, choosing from the context menu Settings – Basic – Angle between 0^{o} and 180^{o}.
13.		Insert dynamic text showing the value of internal angles, eg. typing in the window Editing α = α (α choose from the scroll list Symbols αβγ, whereas α choose from the scroll list Objects) Do the same with the β and γ angles.
14.		Type in Input Bar: sum= α+β+γ to calculate the sum of angles in the triangle.
15.		Insert dynamic text showing the angle sum typing in the box Editing. α+β+γ = α+β+γ=sum (sum, α,β,γ choose from the scroll list Objects , whereas α,β,γ from the scroll list Symbols).
16.		• Format the objects and place the texts, • *a* and *b* slider – change the colour to the same as rotated triangle, • Adjust the colours of the texts with the value of angles to the angles in triangle, • Change the sliders values and observe the impact on the angles placement.

The lenght of square side



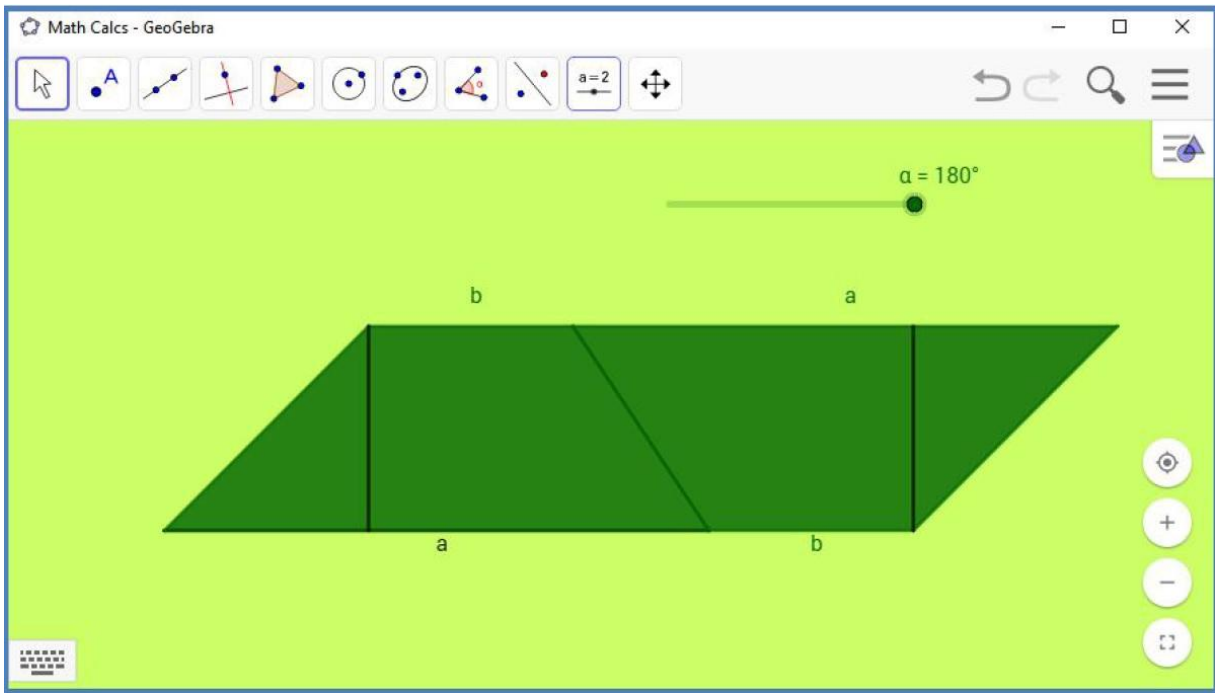
- Turn on the grid and Input bar

No	TOOL	PROCESS STEPS
1.	ABC	Insert text: The length of square side
2.		Insert slider *a*, interval between 1 and 20. Enter Slider Settings – Basic card and in *Caption* box type in: *the length of side a* and change the option Show label into *Caption*.
3.		Draw a segment from point A with length *a*.
4.		Draw a circle with the centre B and radius *a*.
5.		Draw a perpendicular line to AB passing through point B
6.		Define the intersection of circle and line using the *Intersect* tool; point C will be created
7.		Draw a parallel line to AB passing through point C

8.		Draw a perpendicular line to AB that pass through point A
9.		Define the intersection of both lines – point D; this way all the polygon vertices were defined
10.		Using the *Polygon* tool draw a square ABCD
11.		In the input bar type in P=a^2  Set up a slider value to 4 (to make square area bigger). Hide unnecessary objects.
12.		Insert text P=P and move it into figure area. Attention! Choose P from the scroll list Advanced-Objects  Attention! To see the text defining the area value inside the polygon choose *Midpoint* tool  and click on B and D points (point E will be created). Enter the Settings of the text P=P and in the *Position* tab choose point E as your Starting Point.
13.		Insert text $a = \sqrt{P}$ Tip: Advanced – LaTeX Formula, choose $\sqrt{x}$, change x into P.
14.		Using the tool *Check Box to Show/Hide objects* join the option box with appropriate text (In the Caption box type in: *Give the formula*; then choose appropriate text - $a = \sqrt{P}$).
15.		Insert dynamic text: $a = \sqrt{P} = a$ (P and a choose from Objects , whereas square root ($\sqrt{x}$) from LaTeX Formula
16.		Using the tool *Check Box to Show/Hide objects* join the option box with appropriate text (In the Caption box type in: *give the the length of the square side knowing its area* then choose *text4*).
17.		In the Input bar type in: Ob=4a and click Enter

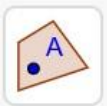
18.		Insert dynamic text: Ob.=4a=Ob.
19.		Using the tool *Check Box to Show/Hide objects* join the option box with appropriate text (In the Caption box type in: *calculate the square perimeter* then choose *text5*).
20.		Format texts and objects.

Trapeze area visualisation



Create the applet to illustrate trapeze area

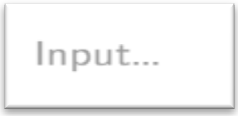
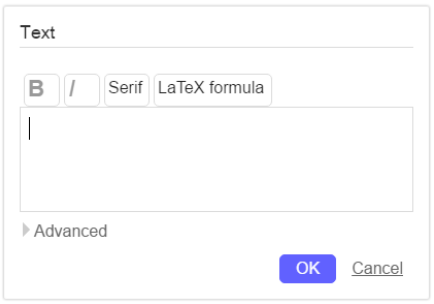
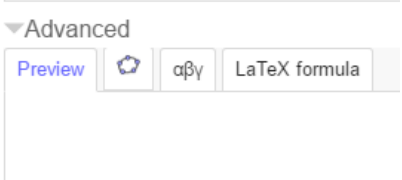
- Open a New Geogebra file
- Hide axes and grid
- Change the setting of the Objects label (choose Labelling from the menu Options, and then Only New Points).
- Follow the steps:

No	TOOL	PROCESS STEPS
1.		Using Segment tool construct AB segment.
2.		Insert point C that isn't collinear with points A and B.
3.		Using *Paraller Line* tool draw a parallel line to AB passing through point C
4.		Use the *Point on the object* tool and mark point D on the created parallel line.
5.		Construct trapeze ABCD
6.		Fix points A, B, C, D – click on the right mouse button, choose Settings and in tab Basic mark Fix Object

7.		Draw a perpendicular line to the line containing AB segment passing through vertex C
8.		Define the intersection point of created perpendicular line and segment AB. Point E will be created.
9.		Hide the parallel to AB line and the line containing its height (right button on the mouse, uncheck Show object).
10.		Draw segment CE, which will be the trapeze height
11.		Define the midpoint of the trapeze BD side. Point F will be created.
12.		Create a slider for angle α MIN 0°MAX 180° Increment 1°
13.		Using the tool *Rotate around point*, rotate the trapeze ABCD and its height to α angle around point F. To do it, click on the trapeze and then point F. **Remember insert α in Angle Box.** Hide the labels of all the points.
14.		Set the same labels to all corresponding trapezoidal bases Use *Caption* box.

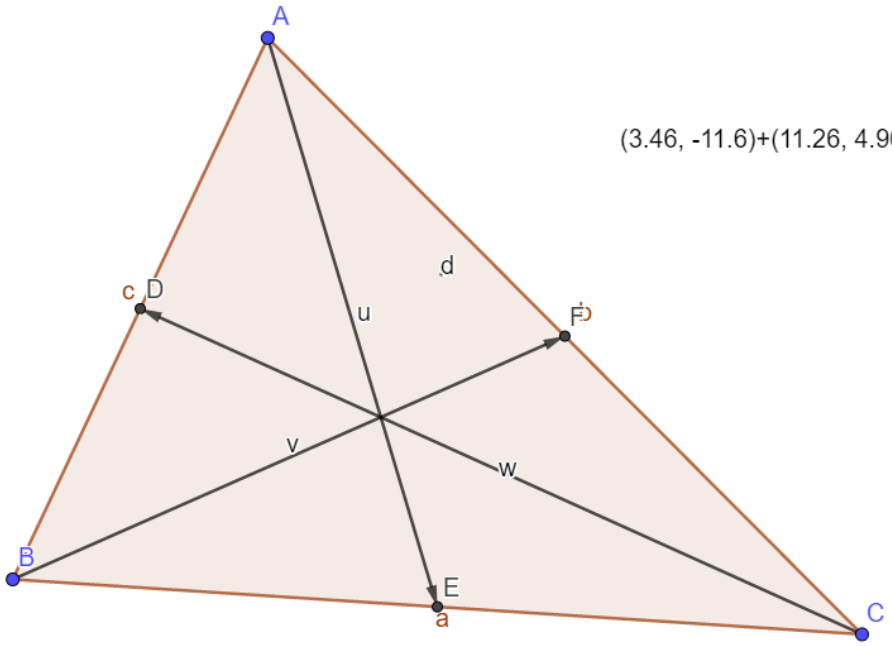
EUROGEBRA WORKSHEET

THE SUM OF THE VECTORS UPON THE MEDIANS OF A TRINGLE IS ALWAYS A ZERO VECTOR

MENU	TOOL	PROCESS STEPS
	 Polygon	**Draw a random trangle ABC**
	 Midpoint or Centre	**Construct the midpoints D,E,F of the triangle sides AB,BC,CA**
	 Vector	**Construct the vectors u,v,w from points AE,BF,CD**
		**Write to the input line the sum: u+v+w and find that equals to zero vector**
	ABC Text	 ***Click ->Advanced**  ***Click ->Geogebra Symbol**

***From the list below click the letters u,v,w,d and with + and = form the equation:**

u + v + w = d

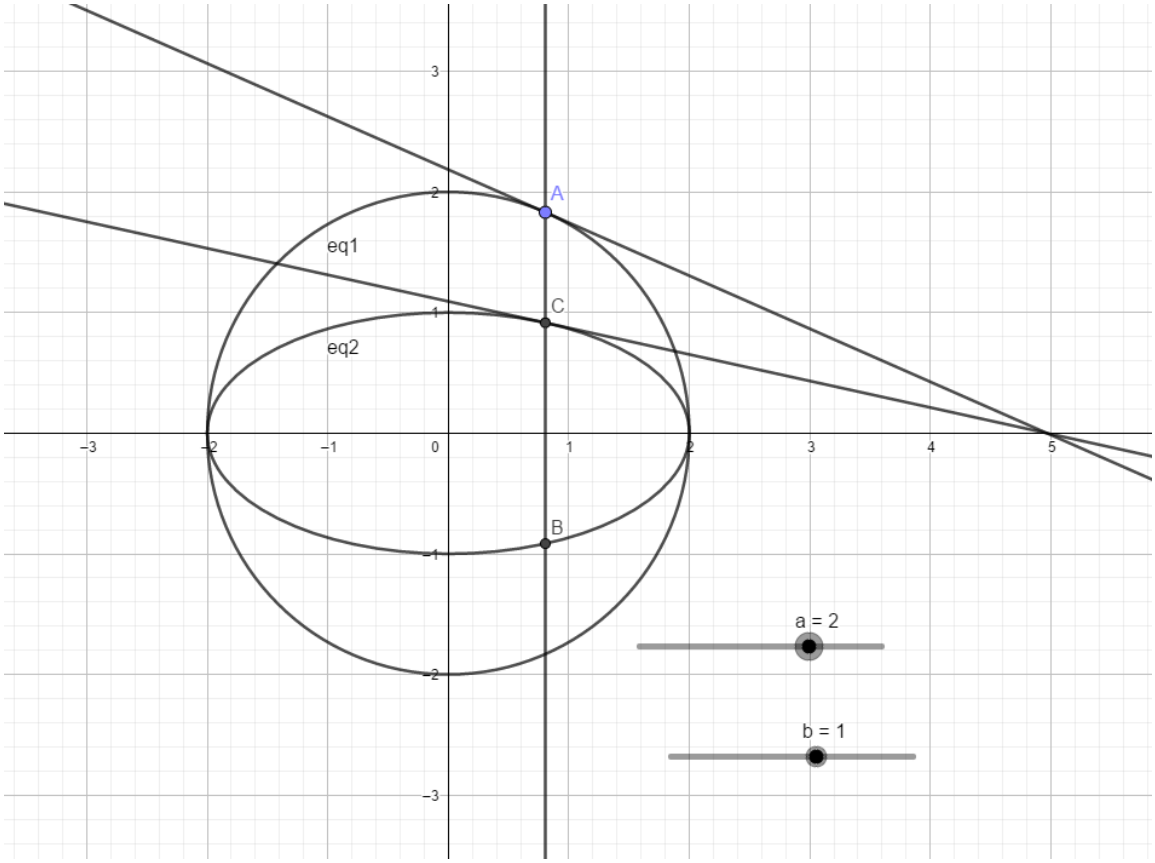


(3.46, -11.6)+(11.26, 4.96)+(-14.72, 6.64)=(0, 0)

EUROGEBRA WORKSHEET

THE TANGENTS OF CIRCLE $x^2 + y^2 = a^2$ AND ELLIPSE $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ AT POINTS WITH SAME X COORDINATE, MEET ON X-AXIS

MENU	TOOL	PROCESS STEPS
	 Slider	• Click to the tool and then click anywhere on the plane to create slider a. At the popup menu click OK. • The same to create slider b.
		Write the following equations: • $x^2 + y^2 = a^2$ • $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Pressing Enter each time)
	Point	Place a random Point A on the circle
	 Perpendicular Line	Click on point A and x axis
	 Intersect	Click on perpendicular and ellipse to find intersect points B and C
	 Tangents	Create the tangents of the circle and the ellipse on points A and C
We can see that the tangents meet on the x-axis. Even if we change the parameters moving the sliders the same happens. Why? Can you prove it?		


A
C
B
eq1
eq2
a = 2
b = 1

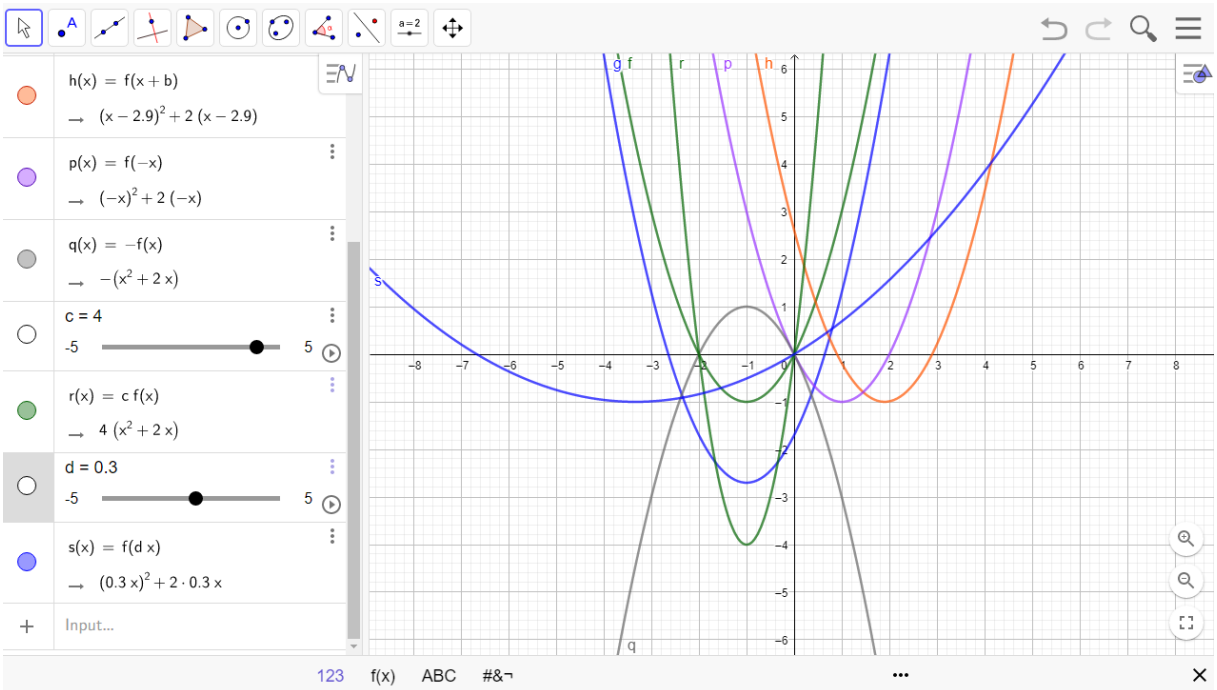
EUROGEBRA WORKSHEET No

TITLE : Transformation of functions

MENU	TOOL	PROCESS STEPS
		In the input bar type in f(x) =x^2 + 2x
		In the input bar type in f(x)+a . Move the slider a and answer the questions below.
		In the input bar type in f(x+b). Move the slider b and answer the questions below.
		In the input bar type in f(-x). Answer the questions below.
		In the input bar type in –f(x). Answer the questions below.
		In the input bar type in c*f(x). Answer the questions below
		Create a slider d by typing d in the input bar.
		In the input bar type in f(d*x). Answer the questions below.

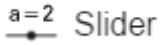
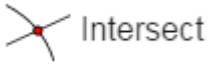
Questions:

1. Describe what happens to the graph when you adjust the slider a for f(x) +a. What is this type of transformation called? The graph of y=f(x)+a is a translation of the graph y=f(x) by the vector(0,a). The graph moves in the y direction by a units.
2. Describe what happens to the graph when you adjust the slider b for f(x+b) . What is this type of transformation called? The graph of y=f(x+b) is a translation of the graph y=f(x) by the vector(-b,0). The graph moves in the opposite x direction by b units. If b is 2 then the graph will be translated by 2 units in the negative x direction.
3. Describe what happens to the graph when you type in f(-x). What is this type of transformation called? This is a reflection of the graph f(x) in the y-axis
4. Describe what happens to the graph when you type in –f(x). What is this type of transformation called? This is a reflection of the graph f(x) in the x-axis
5. Describe what happens to the graph when you adjust the slider c for c* f(x). What is this type of transformation called? The graph of y=c*f(x) is a stretch of the graph y=f(x) in the y direction by a scale factor of c . The y coordinates are increased by a factor of c. The x coordinates are unchanged.
6. Describe what happens to the graph when you adjust the slider d for f(d.x) . What is this type of transformation called? The graph of y=f(d*x) is a stretch of the graph y=f(x) in the x direction by a scale factor of 1/d. The x coordinates are increased by a factor of 1/d. The y coordinates are unchanged.

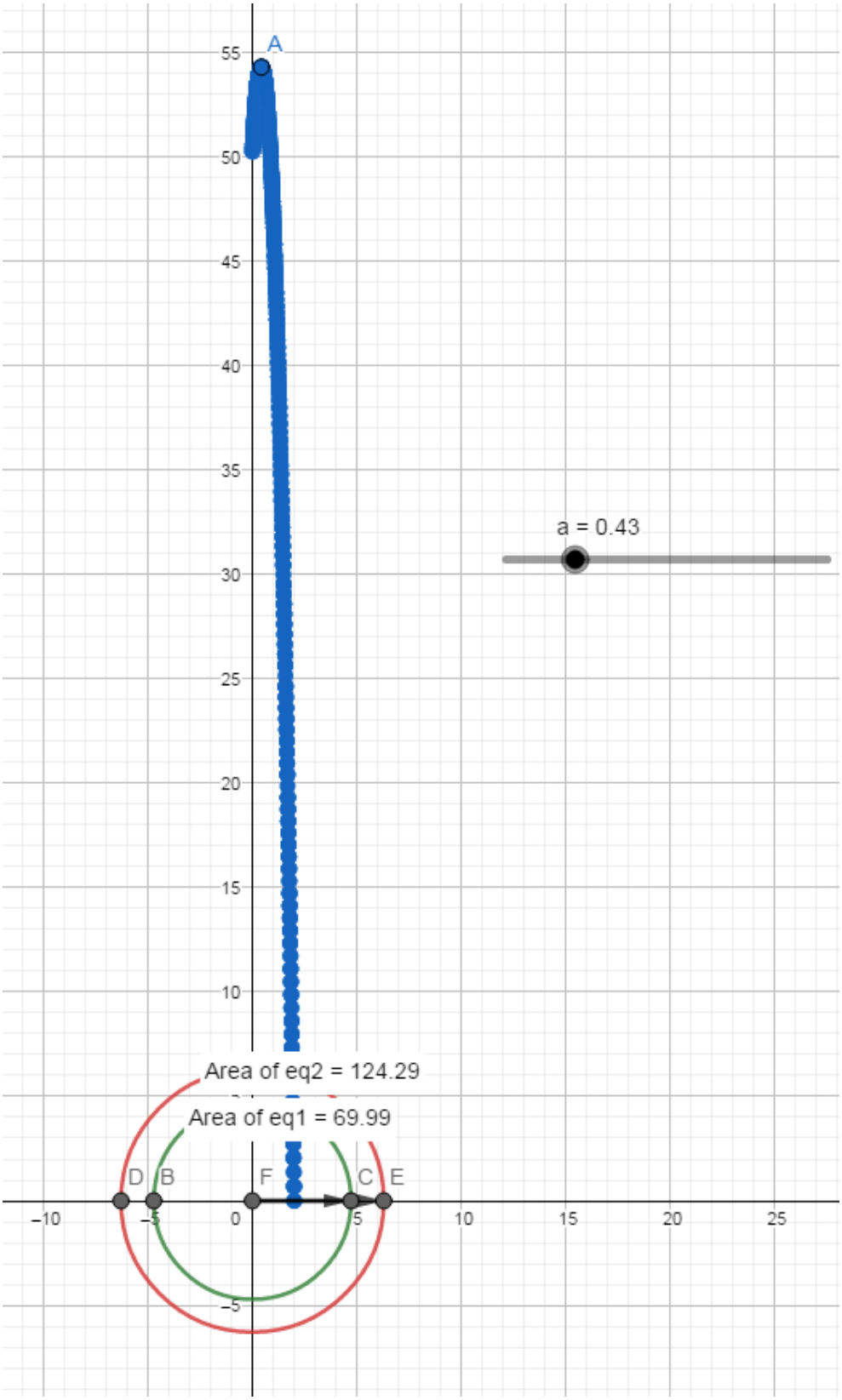


EUROGEBRA WORKSHEET

VARIABLE CIRCULAR RING AND ITS VECTOR RAYS

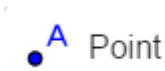
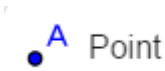
MENU	TOOL	PROCESS STEPS
		Cilck on thw geogebra board to define a slider „a“ , set min = 0 and max = 2
Write in the input cell the equation „$x^2 + y^2 = (3 + 4\cdot a)^2$” to create a circle with centre the O(0, 0) point and ray $r_1 = 3 + 4\cdot a$.		
Write in the input cell the equation „$x^2 + y^2 = (5 + 3\cdot a)^2$” to create a circle with centre the O(0, 0) point and ray $r_2 = 5 + 3\cdot a$.		
		Click on the outer circle to measure its area (b = area(eq2)
		Click on the inner circle to measure its area (c = area(eq1)
Write in the input cell the subtraction „b – c“ and the conclusion will be the variable “d” („d“ is difference between the two areas)		
		Select the A point and insert in the input cell the coordinates (a, d)
Right click on „A point“ to check the „show trace“ box.		
		Click on the xAxis and inner circle to see the intersection points B and C.

	 Intersect	**Click on the xAxis and outer circle to see the intersection points D and E.**
	 Intersect	**Click on the xAxis and yAxis to see the intersection point F.**
	 Vector	**Click on the F and C points to draw the u vector The u vector is the r_1 ray**
	 Vector	**Click on the F and E points to draw the v vector The v vector is the r_2 ray**
Click the "animation" button of the "a" slider to see the change in the area "d" of the circular ring as a function of the slider "a"		
Excercise : Let "d" be the area of the circular ring. Assume that at time t = a = 0 is r_1 = 3 cm and r_2 = 5 cm and the radius r_1 increases at a constant rate of 4 cm / sec, while r_2 at a rate of 3 cm / sec.		
1 st task : when the area of the circular ring will be zeroed.		
2nd task : when the area of the circular ring will be maximized.		
3rd task : What is the graph of the function that runs through point A.		


A
a = 0.43
Area of eq2 = 124.29
Area of eq1 = 69.99
D
B
F
C
E
55
50
45
40
35
30
25
20
15
10
−5
−10
0
5
10
15
20
25

EUROGEBRA WORKSHEET

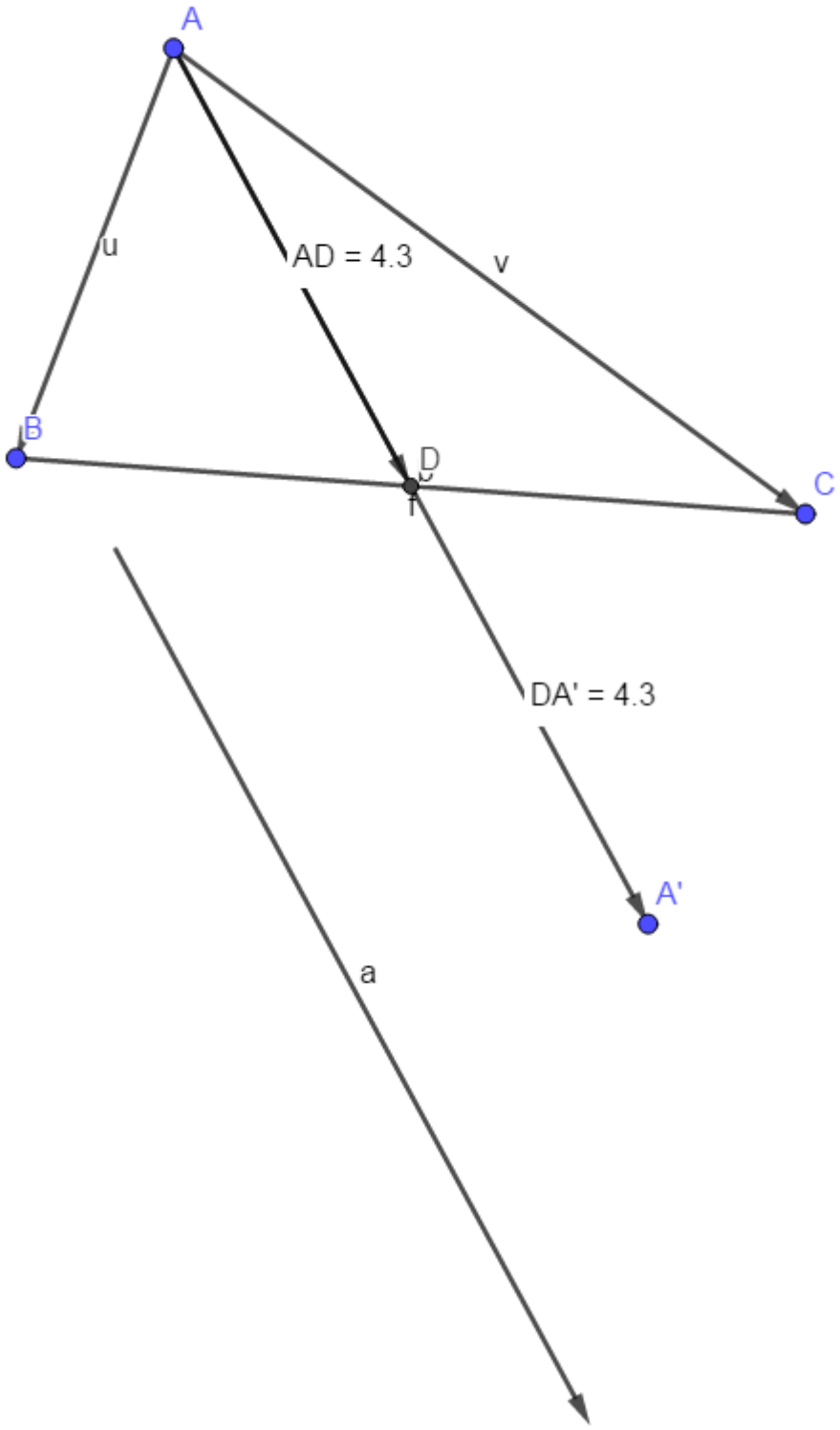
TITLE : VECTOR MID – SECTION RADIUS

MENU	TOOL	PROCESS STEPS
		**Click on the board to see the A point**
		**Click on the board to see the B point**
		**Do the same , to see the C point**
	 Vector	**Select the A point and then the B point to draw the u vector**
	 Vector	**Select the A point and then the C point to draw the v vector**
	 Segment	**Click on the B and C points for the BC segment**
	 Midpoint or Centre	**Click on the B and C points to find the midpoint D of the BC segment**
	 Vector	**Select the A point and then the D point to draw the w vector**

Note that the w vector is the

vector mid – section radius

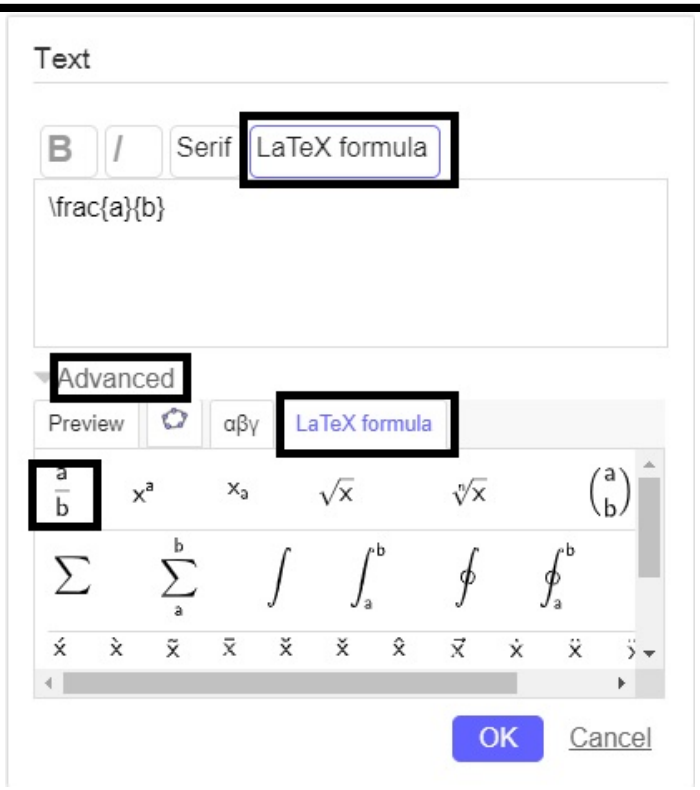
write in the „input“ cell „u + v“ and the „a“ vector will appear in the geogebra board		
Note that the a vector is the **vector sum of u and v vectors**		
	 Vector from Point	Click on A point and then the a vector to draw the b vector (b = a) The b = AA′
	 Distance or Length	Click the A and D points to count the distance AD
	 Distance or Length	Click the D and A′ points to count the distance DA′
1 st task : what do you notice about the two distances		
2nd task : i) Which is the relation between the w and a vectors ? **ii) prove that** $w = \frac{1}{2}$ **a.**		
3rd task : i) repeate the same procedure if the D point is internal point of segment BC such as $DA = \frac{2}{3} DB$ **,** **ii) prove then** $w = \frac{3 \cdot u + 2 \cdot v}{3+2}$		
iii) repeate the same procedure if the D point is internal point of segment BC such as $DA = \frac{m}{n} DB$ **,** $m > 0,\ n > 0$ **ii) prove then** $w = \frac{n \cdot u + m \cdot v}{n+m}$		


A
u
AD = 4.3
v
B
D
C
DA' = 4.3
A'
a

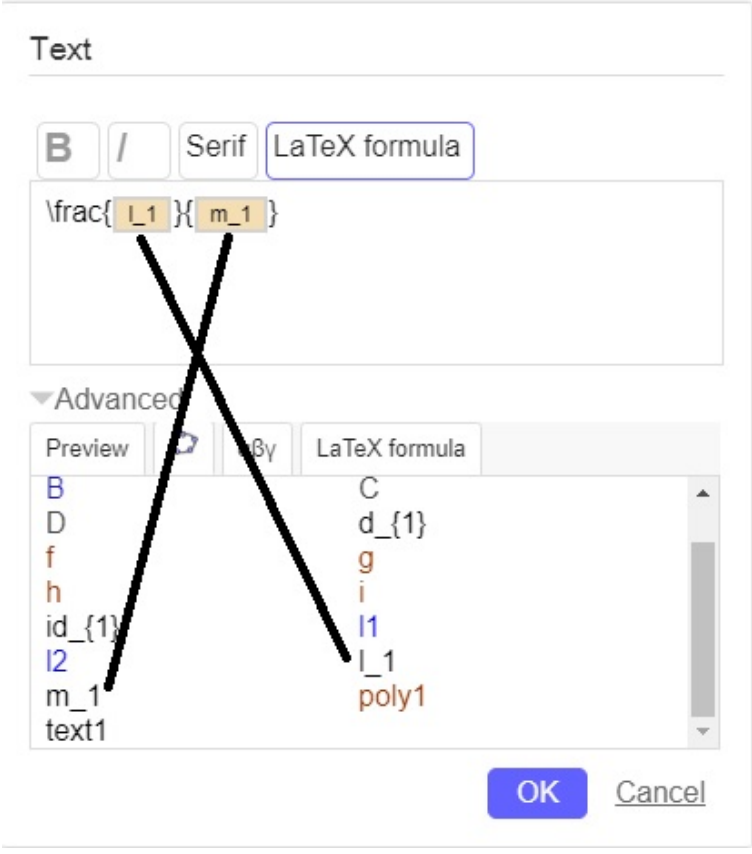
EUROGEBRA WORKSHEET

VISUALISATION OF COMMON (SIMPLE) FRACTIONS

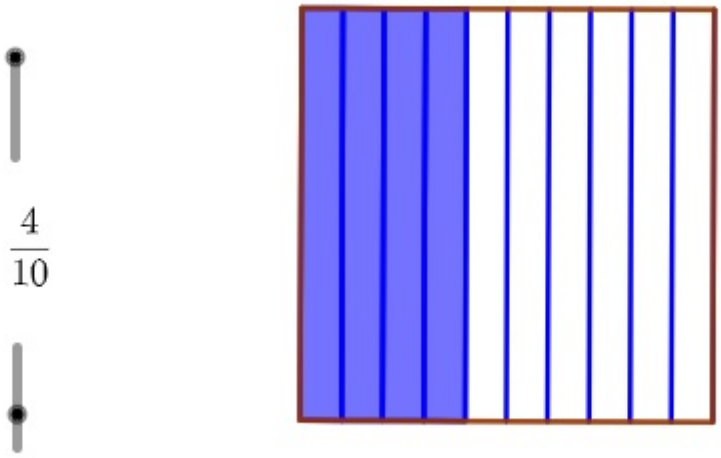
MENU	TOOL	PROCESS STEPS
	 Regular Polygon	Draw a square *ABCD*
	Slider	Create a vertical slider *m_1* MIN 1, MAX 10, Increment: 1 That's the denominator in our fraction.
	 Slider	Create a vertical slider *l_1* MIN 1, MAX m_1, Increment: 1 That's the numerator in our fraction.
		In the input bar type in: d_1=(B-A)/m_1 Thats the width of a rectangle. Hide the vector.
		Create a sequence of rectangles by typing in the input bar: *Sequence(Polygon(A+(i-1)*d_1, A+i*d_1, D+i*d_1, D+(i-1)*d_1),i,1,l_1)* In *Settings* of this object change the colour to blue and set *Opacity* to 50%.
		Create a sequence of segments by typing in the input bar: Sequence(Segment(A+i*d_1, D+i*d_1),i,1,m_1) In *Settings* of this object change the colour to blue.
	ABC Text	Insert a dynamic text showing the value of our fraction



In place of *a* and *b* put our numerator (*l_1*) and denominator (*m_1*) from the *objects tab*



End Result

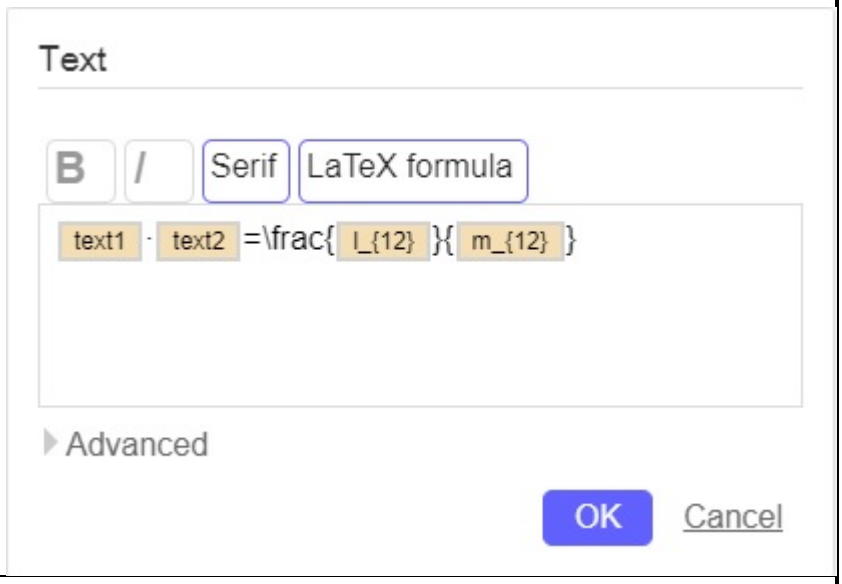
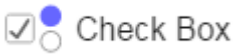


follow-up exercise

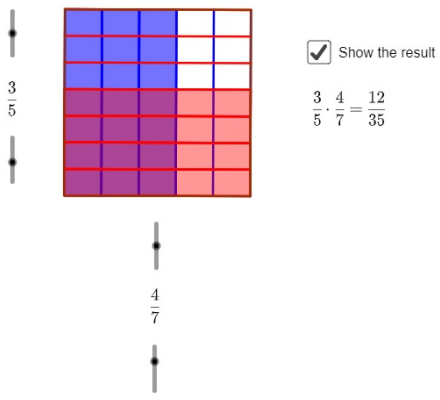
multiplication of common fractions

additions to the previous construction:

MENU	TOOL	PROCESS STEPS
a=2	a=2 Slider	Create a vertical slider *m_2* MIN 1, MAX 10, Increment: 1
a=2	a=2 Slider	Create a vertical slider *l_1* MIN 1, MAX m_2, Increment: 1
		In the input bar type in: d_2=(D-A)/m_2 Hide the vector.
		Create a sequence of rectangles by typing in the input bar: *Sequence(Polygon(A+(i-1)*d_2, A+i*d_2, B+i*d_2, B+(i-1)*d_2),i,1,l_2)* In *Settings* of this object change the colour to red and set *Opacity* to 50%. Create a sequence of segments by typing in the input bar:

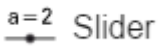
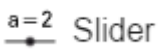
		*Sequence(Segment(A+i*d_2, B+i*d_2),i,1,m_2)* In *Settings* of this object change the colour to blue.
		In the input bar type in: *l_12=l_1*l_2*
		In the input bar type in: m_12=m_1*m_2
		Insert a dynamic text showing the result of multiplication of both fractions: 
		Create a check box that will show/hide the result of multiplication. Choose *text3* as an object to hide/show.

End Result



EUROGEBRA WORKSHEET

VISUALISATION OF INTERVALS

MENU	TOOL	PROCESS STEPS
		Insert slider *a*, which will set the starting point of the interval (interval between -10 and 10; increment 0.1)
		Insert slider *b*, which will set the ending point of the interval (interval between *a*+0.1 and 10; increment 0.1)
		In the *input bar* type in: *a<x<b.* This will create the inequality *c.* Go into *Settings* of this object, click on the *Style* tab and select *show on x-axis.* Set the *Line thickness* to 7. Set the colour to blue (in the *Colour* tab).
		In the *input bar* type in: g=(a+b)/2
		In the *input bar type in: G=(g,1)* This will create a point that will serve as a *Starting Point* for the description of the interval.
		Insert a dynamic text that will describe the inequality (*text1*). Go into *Settings* and change the colour to blue. Go to the *Position* tab and set *Starting Point* to *G.*
		Similarly create: • inequality a<x<=b and *text2* which describes it (use the red colour) • inequality a<=x<b and *text3* which describes it (use the green colour) • inequality a<=x<=b and *text4* which describes it (use the violet/purple colour)

a=2	Check Box	Insert a check box *h* Caption: (a,b) Objects to hide/show: inequality *c* and *text1*
a=2	Check Box	Similarly create check boxes *i*, *j* and *k* that will show/hide the other inequalities and their descriptions
		Go to *Settings* of the check box *h,* choose *Scripting* tab and type in: *i=false* *j=false* *k=false* Thanks to this, checking the *h* check box will uncheck all the others check boxes, so only one inequality will be shown at all times. (inequality *c* in this case*)*.
		Similarly type in the scripts for the other check boxes (*i,j* and *k)*. Remember to set the value to false for every other check box, except the one you are currently working on.
a=2	ABC Text	Insert text: *Choose the type of the interval:*

End result: