



Parabolas Trigonometry





Introduction:

These worksheets were created within the Erasmus + project, Eurogebra.

Worksheets are in the field of mathematics and use the Geogebra program for individual mathematical tasks. The purpose is to use the program when teaching and explaining problems in mathematics and thus to approach the issue more clearly. Worksheets are in the form of specific instructions and tools that will guide us to solve various tasks. In this way, students will get closer to a better understanding and modification of the given examples. Individual groups of worksheets can be combined with each other and create new subgroups according to the issues discussed. Some examples are followed by the solution of examples and free sheets for student notes.

Worksheets respect pedagogical documents related to the subject of mathematics. When working with worksheets, it is necessary to cooperate with teachers and coordinate their work.

In terms of content, we focused on geometric examples, where you can effectively solve problems and modify them in various ways. By formulating the tasks, we lead the students to understand the assigned tasks and to solve the tasks according to the individual steps in the worksheets.





COMPLETED SQUARE FORM

MENU	TOOL		PROCESS STEPS
			In the input bar enter y=x^2+bx+c
	$f: y = x^{2} + 1 x + 0.1$ $p = 1$ -5	5 Spectral S	colour of the function by te clicking on the three dots and no going to settings
			In the input bar enter y=(x+p)^2+q
	<pre>g: y = (x - 1.1)² - 0.8 + Input</pre>	Solve Special Points Duplicate input Delete Settings	Change the colour of the function by clicking on the three dots and going to settings
\bigwedge	Turning poin	t	Click on the turning point function





- 1. Set b=2and c=3 by moving the slider. What is the equation of the function $y=x^2+2x+3$
- 2. Set p=1 and q=2 by moving the slider. What is the equation of the function $\frac{y}{(x+1)^2}$ +2
- The two graphs will now coincide. What does this tell you about the two equations?They are the same. When you expand the brackets and simplify the equation will be the same as 1.
- 4. What is the minimum point on the graph 2?Can you generalise?Can you explain why?Use the turning point function to find the minimum. (-1,2) The minimum point would be (-p,q) This is because of transformations of graphs.
- Is there a relationship so that two graphs will always be the same even though you change the values?Yes if you halve the coefficient of b this will give you the value of p. You then have to square p and work out what you have to add or subtract to get c. This will be the value of q.
- What is the line of symmetry of the curve? Can you generalise your solution? It would be the line x=-p
- 7. What other technique can be used to find the mimimum point on a quadratic? Use differentiation and set the first derivative equal to zero. Solve to find x and substitute back into original equation to find y. Much easier to use the above relationships but it works.



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PARABOLA STRING AND TANGENT

MENU	TOOL	PROCESS STEPS
a=2	a=2 Slider	Cilck on the geogebra board to define a slider "a" , set min = - 3 and max = 3.
a=2	a=2 Slider	Cilck on the geogebra board to define a slider "b" , set min = - 3 and max = 3.
	Vrite in the input cell the function "x ² + a·x The coefficients "a" and "b" of the fuction	-
• ^A	• ^A Point	Select the A point and insert in the input cell the coordinates (a, f(a)). The A point is a point of the parabola.
• ^A	• ^A Point	Select the B point and insert in the input cell the coordinates (b, f(b)). The B point is another point of the parabola.
	🦯 Segment	Click on the A and B points to draw the segment AB. This segment is the string AB of the parabola.
• ^A	• ^A Point	Select the C point and insert in the input cell the coordinates $(\frac{a+b}{2}, f(\frac{a+b}{2})).$ The C point is also a point of the parabola.
+	Tangents	Click on the C point and then the parabola to draw the tangent of the parabola at C point.
1st task : What is the relation of the string and the tangent ?		





~	Line	Click on the A and B points to draw the AB line.
4	Slope	Click on the AB line to measure its slope.
4	Slope	Click on the tangent to measure its slope.
2st task : What is the relationship between the two slopes and how is this related to the result of the first task ?		
3rd task : Could you formulate a rule for the above construction and prove it ?		







QUADRATIC FUNCTION

MENU	TOOL	PROCESS STEPS
	+	In the <i>Algebra</i> view type in: f(x)=ax^2+bx+c GeoGebra will automatically create sliders for the <i>a,b</i> and <i>c</i> coefficients. Hide the label of the graph.
		Create a point on the graph. In the <i>Settings->Advanced</i> tab set <i>Dynamic Colours</i> as following: Dynamic Colours
		Red:
	Point on Object	y(A) > 0
Ľ		Green:
		y(A) ≟ 0
		Blue:
		y(A) < 0
	+	In the Algebra view type in: B=(0,y(A))
	+	In the Algebra view type in: <i>C=(x(A),0)</i>
	🥕 Segment	Draw segments AB and AC. In the Settings->Style tab change their Line Style to a dotted one. Hide their labels. For the vertical segment turn on the Show trace option
		Set <i>Dynamic Colours</i> for the <i>AB</i> and <i>AC</i> segments:





		Dynamic Colours
		Red:
		y(A) > 0
		Green:
		y(A) ≟ 0
		Blue:
		y(A) < 0
		Insert a text:
		y_A>0
		Change its colour to red and set a
a=2	ABC Text	condition to show the object:
\square		y(A)>0
		Set the <i>Starting Point</i> of the object to: A
		(in the <i>Settings->Position</i> tab)
		Insert a text:
		y_A<0
a=2	ABC Text	Change its colour to blue and set a
	ABC Text	condition to show the object:
		y(A)< 0
		Set the <i>Starting Point</i> of the object to: A
		(in the <i>Settings->Position</i> tab)
	 	Using the Intersect tool find intersections
•^	X Intersect	between the graph and the xAxis.
	,	Set labels for them: x_1=%x and x_2=%x







QUADRATIC INEQUALITY

MENU	TOOL	PROCESS STEPS
a=2	a=2 Slider	In the <i>Graphics2</i> view insert sliders <i>a, b</i> and <i>c.</i> Increment 0.1 (use them to change the values of the coefficients)
	+	In the <i>Algebra</i> view type in: f(x) = ax^2 + bx + c
	+	In the <i>Algebra</i> view type in: g(x) = polynomial(f) The polynomial command will sort the formula and show it in a clearer way. You can hide the f function - we`ll be using g instead.
a=2	ABC Text	In the <i>Graphics</i> view insert a dynamic text: <i>f(x) = g</i> (choose <i>g</i> from the list of objects)
	+	In the Algebra view type in: d: $y > 0 \land y < f(x)$ You`ll create an area d, which is a set of points that satisfy both inequalities in the conjunction.Go to Settings of this object, in the Colour tab change it to red, in the Style tab: - set Line Thickness to 0 - set Line Opacity to 100 - change Filling to cross-hatching - set Angle to 0Similarly, in the Algebra view type in e: $y < 0 \land y > f(x)$
	+	Change the <i>Settings</i> just like in the previous step, except this time choose blue colour and <i>hatching Filling</i> .
a=2	ABC Text	In the <i>Graphics2</i> view insert a dynamic text: g > 0 (choose g from the list of objects)



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a=2	ABC Text	In the <i>Graphics2</i> view insert a dynamic text: g < 0 (choose g from the list of objects)
a=2	Check Box	In the <i>Graphics2</i> view insert (next to text2) a check box <i>h</i> that will hide/show the area <i>d</i>
a=2	Check Box	In the <i>Graphics2</i> view insert (next to text3) a check box <i>i</i> that will hide/show the area <i>e</i>
		In the Settings of the check box h: - hide lebel - in Scripting tab type in: i=false (thanks to this checking h will uncheck i) In the Settings of the check box i: - hide lebel - in the Scripting tab type in: h=false (thanks to this checking i will uncheck h) In the Settings of sliders a, b and c go to Scripting->On update and type in: i=false h=false Thanks to this - moving any of the sliders (changing coefficients) will also uncheck both check boxes.
		Hide the <i>Algebra</i> view







TRANSFORMATION OF FUNCTIONS

MENU	TOOL	PROCESS STEPS
		In the input bar type in f(x) =x^2 + 2x
		In the input bar type in f(x)+a . Move the slider a and answer the questions below.
		In the input bar type in f(x+b). Move the slider b and answer the questions below.
		In the input bar type in f(-x). Answer the questions below.
		In the input bar type in -f(x). Answer the questions below.
		In the input bar type in c*f(x). Answer the questions below
		Create a slider d by typing d in the input bar.
		In the input bar type in f(d*x). Answer the questions below.





- Describe what happens to the graph when you adjust the slider a for f(x) +a. What is this type of transformation called? The graph of y=f(x)+a is a translation of the graph y=f(x) by the vector(0,a). The graph moves in the y direction by a units.
- 2. Describe what happens to the graph when you adjust the slider b for f(x+b). What is this type of transformation called? The graph of y=f(x+b) is a translation of the graph y=f(x) by the vector(-b,0). The graph moves in the opposite x direction by b units. If b is 2 then the graph will be translated by 2 units in the negative x direction.
- 3. Describe what happens to the graph when you type in f(-x). What is this type of transformation called?This is a reflection of the graph f(x) in the y-axis
- 4. Describe what happens to the graph when you type in -f(x). What is this type of transformation called? This is a reflection of the graph f(x) in the x-axis
- 5. Describe what happens to the graph when you adjust the slider c for c* f(x). What is this type of transformation called?The graph of y=c*f(x) is a stretch of the graph y=f(x) in the y direction by a scale factor of c. The y coordinates are increased by a factor of c. The x coordinates are unchanged.
- 6. Describe what happens to the graph when you adjust the slider d for f(d.x). What is this type of transformation called?The graph of y=f(d*x) is a stretch of the graph y=f(x) in the x direction by a scale factor of 1/d. The x coordinates are increased by a factor of 1/d. The y coordinates are unchanged.









VISUALISATION AND PROPERTIES OF TRIGONOMETRIC FUNCTIONS Y=ASIN(WX), Y=ACOS(WX) DEPENDING ON THE A AND W COEFFICIENTS

MENU	TOOL	PROCESS STEPS
		First of all we need to prepare the axes and the grid in the <i>Graphics</i> view: - in the <i>xAxis</i> tab change the distance to $\pi/2$ - in the <i>yAxis</i> tab change the distane to 1 - in the <i>Grid</i> tab change the <i>Grid type</i> to <i>Major Gridlines</i> and set the distance to $x: \pi/6, y: 1/2$
		In the <i>Algebra</i> view type in: functions={sin(x),cos(x)} Make it visible in the <i>Graphics2</i> view. In <i>Settings:</i> - change the <i>Caption</i> to <i>f(x)=</i> - select <i>Draw as a drop-down list</i>
		In the CAS view type in: f(x): = SelectedElement(functions) Choosing an element from the drop-down list will create a f function. The graph will be visible in the Graphics view.
a=2	a=2 Slider	In the <i>Graphics2</i> view insert a slider <i>A</i> . MIN: -4,9 MAX: 5 increment 0.2 (the interval and the increment has been chosen purposedly to avoid <i>0</i> value)
a=2	a=2 Slider	In the <i>Graphics2</i> view insert a slider <i>w.</i> MIN: 0.1 MAX 5 increment 0.1
		In the CAS view type in: A f(w x)





	In the <i>CAS</i> view type in: <i>f_1(x): =\$2</i>
	In the <i>Graphics2</i> view insert text: f_1(x)=\vspace{1cm} \$2
	In the Graphics2 view insert a check box a which will show/hide the text from the previous step. Caption: f_1(x) = A f(wx) Set a condition to show text1 and function f_1: a
to observe how changing the fundame d properties of the function you`ll need	ental/basic period of f_1 affects the formula d to insert few texts and check boxes:
	In the CAS view type in: <i>w</i>
	In the CAS view type in: $\frac{2\pi}{\$4}$
	Insert a text in the Graphics2 view:
	B / Serif LaTeX formula
	T=\frac{2 π}{ω}=\frac{2 π}{ \$4 }={ \$5 }
	Advanced
	Preview 🗘 αβγ LaTeX formula
	$\frac{a}{b}$ x^a x_a \sqrt{x} $\sqrt[n]{x}$
	This text shows how the fundamental/basic period <i>T</i> of <i>f_1 function</i> changes depending on the <i>w coefficient</i>





To illustrate the changes in the fundamental/basic period <i>T</i> we`ll need to create an auxiliary function <i>g</i> , which graph will be limited by the fundamental/basic period of f_1 function		
		In the <i>CAS</i> view type in: g(x): = Function(\$2,0,\$5)
		Make the graph visible in the <i>Graphics</i> view
		In the <i>Graphics2</i> view insert a check box <i>b</i> . Caption: Fundamental/basic period of f_1
a=2	Check Box	In the <i>Settings</i> of <i>text2</i> and the <i>g</i> function set a condition to show those objects: <i>b</i>
		In the <i>Graphics2</i> view insert a text:
		B / Serif LaTeX formula
A gi	raphic illustration of a set of values of t	$ZW_{f_1}= \label{eq:abs(A)} \ ; \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $
		In the <i>Algebra</i> view type in: <i>h= Segment[(0,-abs(A)), (0,abs(A))]</i> Set the colour of the segment to red.
a=2	Check Box	In the <i>Graphics2</i> view insert a check box <i>c</i> Caption: Set of values of the f_1 function In the <i>Settings</i> of <i>text3</i> and the <i>h</i> segment set a condition to show those objects:
		c





In the <i>Settings</i> of the <i>functions</i> object, in
the <i>Scripting->On Update</i> tab type in:
SetValue(A,1)
SetValue(w,1)
SetValue(a,false)
SetValue(b,false)
SetValue(c,false)
Thanks to this changing the formula of <i>f</i>
will set the value of the A and w
coefficients to 1 and will uncheck the a,b
and <i>c</i> check boxes
In the <i>Settings</i> of <i>a,</i> in the <i>Scripts->On</i>
<i>update</i> tab type in:
b=false
c=false
In the <i>Settings</i> of <i>b</i> and <i>c,</i> in the
Scripts->On update tab type in:
a
Set the colours of the corresponding objects. Hide the CAS and Algebra views.







TITLE : Differentiation and of sin and cos functions

MENU	TOOL	PROCESS STEPS
		From the algebra view type in f(x)= sin x
		Type in f'(x).
		From the algebra view type in g(x)= cos x
		Type in gʻ(x).
		From the algebra view type in h(x)= -cos x Type in h'(x)
		From the algebra view type in j(x)= -sinx Type in j'(x)
		From the algebra view type in f(x)= sin 2x





- 1. From the algebra view type in $f(x) = \sin x$. The x scale is in radians
- 2. Type in f'(x). What do you notice? The derivative is
- 3. From the algebra view type in $g(x) = \cos x$. The x scale is in radians
- 4. Type in g'(x). What do you notice? The derivative is
- 5. From the algebra view type in h(x)= -cos x. The x scale is in radians
- 6. Type in h'(x). What do you notice? The derivative is
- 7. From the algebra view type in f(x) = -sinx. The x scale is in radians
- 8. Type in f'(x). What do you notice? The derivative is -

9. Repeat steps 1-8 but instead use 2x in place of x. What do you notice? Can you generalise your solution for differentiating cos nx and the sin nx where n is a positive integer.

- 10. What about if n was a negative integer?
- 11. How would you integrate the functions in steps 1-10?







SUM OF A GEOMETRIC PROGRESSION

MENU	TOOL	PROCESS STEPS
		Type in f(x)=2^x
		Click view and then spreadsheet
		Input 1,2,3,4,5,6,7,8,9,10 in column A by clicking on the bottom right corner and dragging down
		In column b type in f(A1). Fill the rest of the cells by clicking on the bottom right corner and dragging down
		In column c type in sum (B1:B10)
		Answer the questions below
		Change the value of r to a number less than 1 but greater than 0.
		Answer the question below





- 1. The first term is 2. What do you have to do to get to the next term? Multiply by two
- 2. What about the third and forth terms and the nth term? Multiply by two

3. For the function f(x), the first term is defined as a and the multiplier is defined as r. What expression would define the second term? ar

- 6. What about the third and forth terms? ar^2 and ar^3
- 7. What about the nth term? ar^(n-1)
- 8. Define the S as the sum of a series in terms of a,r and n. S = a+ar+....+ar^n-1

9. Multiply S by r. rS= ar+ar^2+...+ar^n

10. Subtract expression from step 9 from step 8 S-rS = a-ar^n

11. Factorise your expression and come up with an expression for S in terms of a,n and r.

S=a(1-r^n)/(1-r)

12. Check to see if this works with n=10, a=2 and r=2. Yes it does

13. Try different geometric sequences. Remember that the graph will be an exponential curve.

14. If you change the number 2 to a number less than 1 but greater than 0 what do you observe? The graph converges.

15. If the value is 0.5 what does this mean? This divides the previous term by 2.

16. What is the sum ? The sum converges to 1.

17. If you were to increase the number of terms to infinity what would happen to r^n if r is 0.5? It would tend to zero.

18. What would be the sum to infinity if r<1? S=a/(1-r)

19. If the value is -0.5 what does this mean? This divides the previous term by 2.

16. What is the sum ? The sum converges to 1.

17. If you were to increase the number of terms to infinity what would happen to r^n if r is 0.5? It would tend to zero.

18. What would be the sum to infinity if 0 < r < 1? S=a/(1-r)

19. What would the sum be if r<0? If you make sure that -1<r<1 why does the sum to infinity formula still work? r^infinity would tend to zero





20. Explain what happens when r<-1. The sequence alternates between a negative number and a positive number and does not converge.











SUM OF AN ARITHMETIC PROGRESSION

MENU	TOOL	PROCESS STEPS
		Type in f(x)=3x+2
		Click view and then spreadsheet
		Input 1,2,3,4,5,6,7,8,9,10 in column A by clicking on the bottom right corner and dragging down
		In column b type in sum (A1:A10)
		In column c type in f(A1). Fill the rest of the cells by clicking on the bottom right corner and dragging down
		In column d type in sum (C1:C10)
		Answer the questions below





- 1. If the first term is defined as n what would be the second term for the sequence 1,2,3....e.t.c.? n+1
- 2. What about the third and forth terms and the nth term? n+2, n+3, n-1
- 3. By reversing the numbers 1-10 and adding them to the original sequence 1-10 what do you get?

1+2+3+4+5+6+7+8+9+10

+

10+9+8+7+6+5+4+3+2+1

= 11 x10

Half your answer to find the sum of 1-10

4. Generalise your answer for n terms. $(\frac{1}{2})n(n+1)$

5. For the function f(x), the first term is defined as a and the difference between terms is defined as d, what expression would define the second term? a+d

6. What about the third and forth terms? a+2d and a+3d

7. What about the nth term? a+(n-1)d

8. Repeat steps 3-4 to find the sum of a arithmetic sequence S = (1/2)n(2a+(n-1)d)

9. The last term can defined as L which is the same as the nth term. Rewrite the sum in terms of a, n and L . S = (1/2)n(a+L)

10. Check to see if the two formulas are correct for different functions. The functions must be linear.



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VISUALISATION OF SUM OF THE ANGLES IN A TRIANGLE

MENU	TOOL	PROCESS STEPS
	Polygon	Draw a triangle ABC
٩	🗸 Angle	Find the value of all angles of the triangle
•	• Midpoint or Center	Find midpoints of segments AC and BC. Mark them as points D and E.
a=2	$\frac{a=2}{\bullet}$ Slider	Create a slider for angle δ 。 MIN 0 MAX 360 INCREMENT 1
a=2	$\frac{a=2}{\bullet}$ Slider	Create a slider for angle ε 。 MIN 0 MAX 360 INCREMENT 1
•	Rotate around Point	Rotate the triangle ABC around D point by δ angle [counterclockwise]
	δ = 180°	Set the δ angle slider to 180
٩	Angle	Find the value of all angles of the rotated triangle
•	Rotate around Point	Rotate the triangle ABC around E point by ε angle [counterclockwise]





	ε = 180°	Set the ε angle slider to 180
4	🗸 Angle	Find the value of all angles of the rotated triangle





BASIC TRIGONOMETRIC FUNCTIONS

MENU	TOOL	PROCESS STEPS
		First of all we need to prepare the axes and the grid: - in the <i>xAxis</i> tab change the distance to $\pi/2$ - in the <i>yAxis</i> tab change the distane to 1 - in the <i>Grid</i> tab change the <i>Grid type</i> to <i>Major Gridlines</i> and set the distance to x: $\pi/6$, y: 1/2
a=2	a=2 Slider	Insert a slider <i>n</i> , that will let you pick one number out of 7 (MIN: 1, MAX: 7, Increment: 1). Make it vertical and change the <i>Width</i> to 50.
		In the CAS view type in: $a := \left\{ -\frac{\sqrt{3}}{2}, -\frac{\sqrt{2}}{2}, -\frac{1}{2}, 0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2} \right\}$
		In the CAS view type in: b: = a(n)
		In the <i>Algebra</i> view type in: functions={sin(x), cos(x)} In <i>Settings:</i> - change the <i>Caption</i> to <i>f(x)=</i> - select <i>Draw as a drop-down list</i>
		In the CAS view type in: f(x):= SelectedElement(functions)
		In the CAS view type in: <i>list1: = {Roots(f(x)-b,-10,10)}</i> That's a list of roots of <i>f(x)-b</i> function in a <-10,10> interval
		In the CAS view type in: Solve(f(x)=b,x) That's a list of all solutions of <i>f(x)=b</i>





		equation, with x as unknown.
		In the <i>Algebra</i> view type in: y=b
		In the CAS view type in: Simplify(Element(a,n)) This chooses nth element from list a, and removes irrationality from it (if present).
		In the <i>CAS</i> view type in: \$3
a=2	ABC Text	Insert the following text in the Graphics view: Vspace{1cm} \$7 = \$6 Preview C αβγ LaTeX formula (empty box)a b f functions g k_1 list1 list2 n text2 Move the text next to the slider n
a=2	ABC Text	Insert the following text in the <i>Graphics</i> view: List of Solutions\\ \vspace{1cm} \$5, k_i ∈ C\\
		Change the colours of corresponding objects. Hide the CAS and Algebra views











TRIGONOMETRIC CIRCLE AND BASIC TRIGONOMETRIC IDENTITIES

Graphics ↓ Show Axes ✓ ∰ Show Grid ∧ No Grid Major Gridlines ✓ Major and Minor Gridlines Polar	Left Click and select Major gridlines
Isometric C: Snap to Grid ✓ C Clear all Traces To Zoom to fit ✿ Settings	
• ^A Point	Create point A(0,0) and B(1,0) and C(0,1)
Circle with Centre	Create circle (A,B)
Perpendicular Line	Create: PerpendicularLine(B, xAxis) and
• ^A Point	Create point D on the circle
Line	Create line AD
Anale	Create Angle(BAD)=α













	$o = \sin^{2}(\alpha) + \cos^{2}(\alpha)$ $r = \frac{\sin(\alpha)}{\cos(\alpha)}$ $s = \frac{\cos(\alpha)}{\sin(\alpha)}$	
ABC Text	Text B Serif LaTEX formula sin^{2} a + cos^{2} a = 0 Advanced rever opv LaTEX formula (empty box) A B C B C <td< th=""><th>Text Image: Senf LateX formula Vrac{sin a }{cos a }= r - Advanced Image: Senf LateX formula Veriew Image: Senf LateX formula Image: Senf LateX formula</th></td<>	Text Image: Senf LateX formula Vrac{sin a }{cos a }= r - Advanced Image: Senf LateX formula Veriew Image: Senf LateX formula Image: Senf LateX formula





