EUROGEBRA WORKSHEETS

Plane geometry III

EUROGEBRA WORKSHEETS

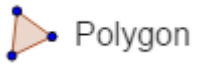
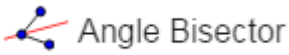
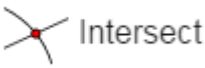
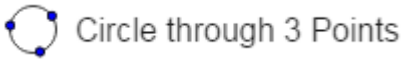
TITLE: PLANE GEOMETRY

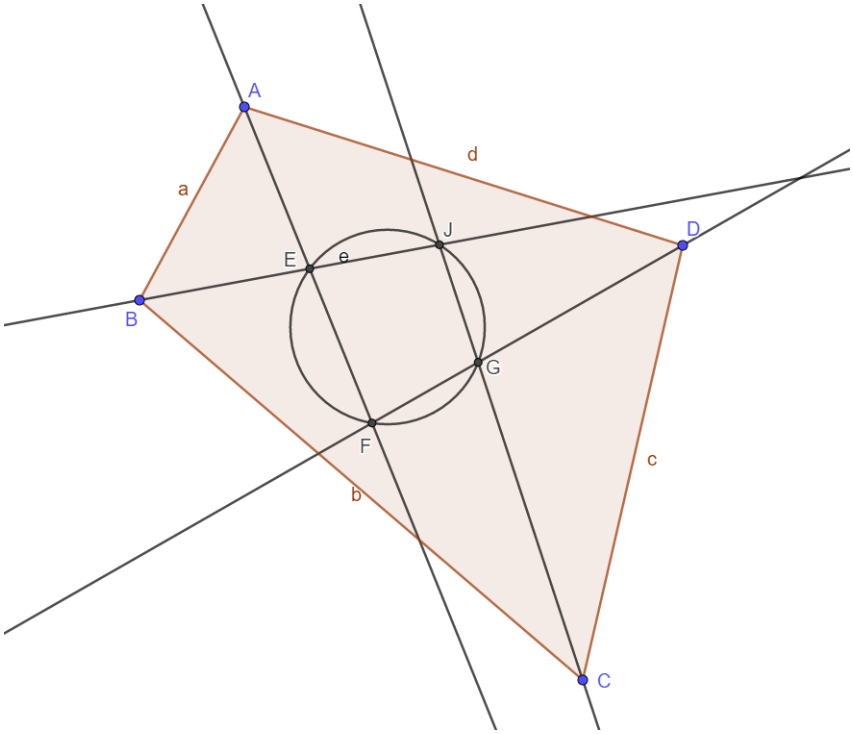
Introduction:

These worksheets were created within the Erasmus + project, Eurogebra.
Worksheets are in the field of mathematics and use the Geogebra program for individual mathematical tasks. The purpose is to use the program when teaching and explaining problems in mathematics and thus to approach the issue more clearly.
Worksheets are in the form of specific instructions and tools that will guide us to solve various tasks. In this way, students will get closer to a better understanding and modification of the given examples. Individual groups of worksheets can be combined with each other and create new subgroups according to the issues discussed. Some examples are followed by the solution of examples and free sheets for student notes.
Worksheets respect pedagogical documents related to the subject of mathematics. When working with worksheets, it is necessary to cooperate with teachers and coordinate their work.
In terms of content, we focused on geometric examples, where you can effectively solve problems and modify them in various ways. By formulating the tasks, we lead the students to understand the assigned tasks and to solve the tasks according to the individual steps in the worksheets.

EUROGEBRA WORKSHEET

THE BISECTORS OF ANY QUADRILATERAL FORM A QUADRILATERAL INSCRIBED IN A CIRCLE

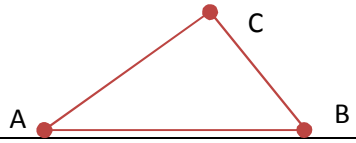
MENU	TOOL	PROCESS STEPS
		**Draw a random quadrilateral ABCD**
		**Construct angle bisector lines from vertex A,B,C and D**
		**Mark the intersections of every two angle bisector lineswith letters E,F,G,J**
		**Click on any 3 vertex of the quadrilateral EFGJ and see that it is inscribed in a circle.**



EUROGEBRA WORKSHEET

CIRCLE IN TRIANGLE

Find the incentre and the inscribed circle of a triangle

MENU	TOOL	PROCESS STEPS
		
	 Angle Bisector	**Constructthe angle bisector of angle BAC.**
	 Angle Bisector	**Construct the angle bisector of angle ABC.**
	 Intersect	**Find the intersection of the two angle bisectors (point D). This is the incentre.**
	 Perpendicular Line	**Construct a line perpendicular to the line AB that goes through point D.**
	 Intersect	**Find the intersection of the perpendicular line and the line AB (point E).**
	 Circle with Center through Point	**Construct a circle with centre at point D and radius DE. This is the inscribed circle.**

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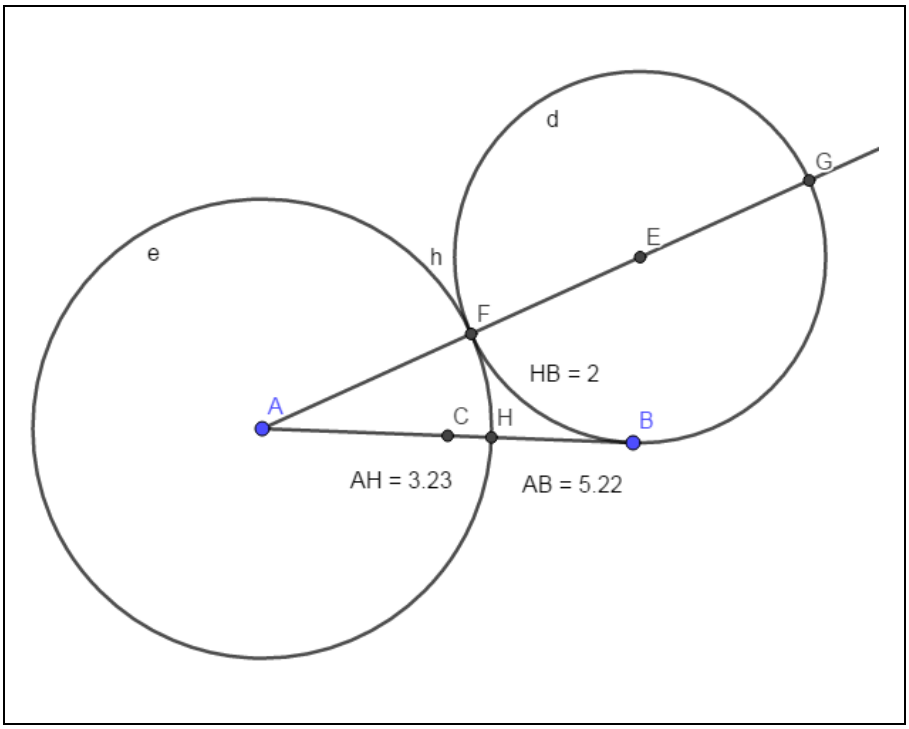
GOLDEN SECTION

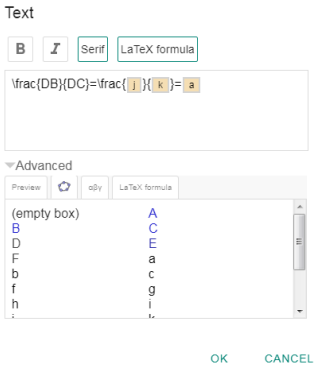
MENU	TOOL	PROCESS STEPS
	 Segment	**Draw a segment AB.**
	 Midpoint or Centre	**Find the midpoint C of the segment AB.**
	 Circle with Centre through Point	**Click the B point to draw the circle with B point as centre and through point C.**
	 Perpendicular Line	**Click point B and AB segment to draw the perpendicular line.**
	 Intersect	**Click the perpendicular line g and the circle B to find the intersection points E and D.**
Left click on the perpendicular line g, then right click and uncheck the „show object“ checkbox. Repeate the steps for the circle B.		
	 Circle with Centre through Point	**Click the E point to draw the circle through point B.**
	 Ray	**Click the A point to draw the ray through E point.**
	 Intersect	**Click the AE ray and the E circle to find the inersection points F and G.**

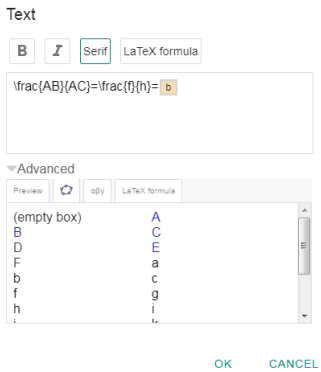
	 Circle with Centre through Point	**Click the A point to draw the circle through point F.**
	 Intersect	**Click the AB segment and the A circle to find the intersection point H.**
	 Distance or Length	**Click the A, B points to messure the AB segment's lenght. Do the same for the AH and HB segments' lenghts.**
Go to the „input" section on the left and insert „$\frac{AB}{AH}$" and then „$\frac{AH}{HB}$" **$\frac{AB}{AH}$ = a and $\frac{AH}{HB}$ = b**		
Question : What do you notice about the two ratios „a" and „b" above. Explain your answer.		



EUROGEBRA WORKSHEET

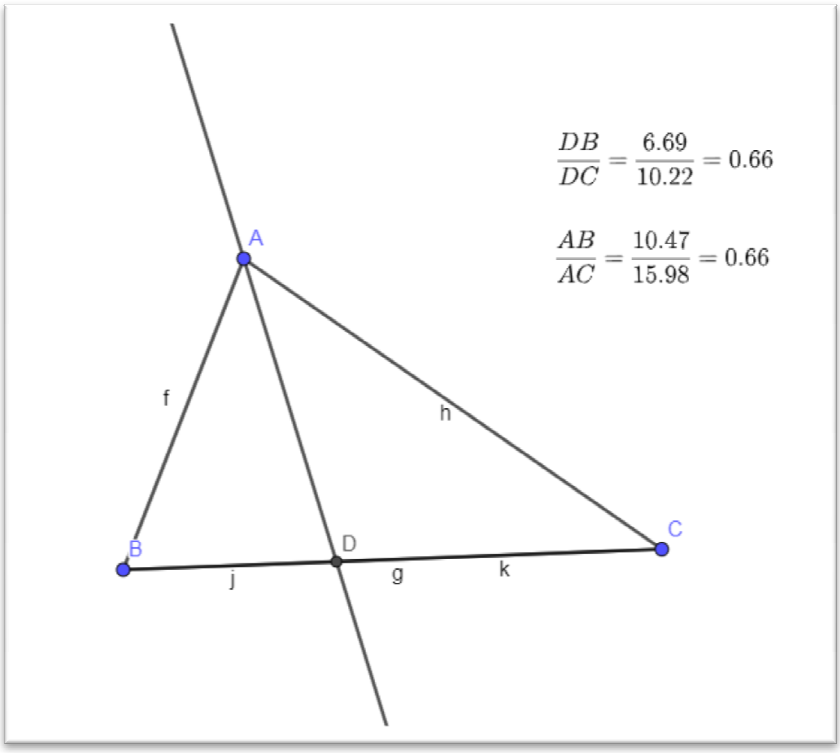
INTERNAL BICECTORS THEOREM

MENU	TOOL	PROCESS STEPS
 Segment	With this buttons create triangle ABC	
 Angle Bisector	Create the angles A bisector clicking B,A,C points (with that order)	
 Intersect	Create the intersect point D of the segment BC and the bisector	
 Segment	Create the segment BD and DC	
	From Algebra section type j/k (creates a) and f/h (creates b)	
ABC Text	From Geometry section, create the following: 	

ABC Text	Similarly create 	

What do you notice ?

Try to discover a similar relation with the help of external bicector of A angle



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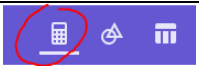
ISOSCELES TRIANGLE

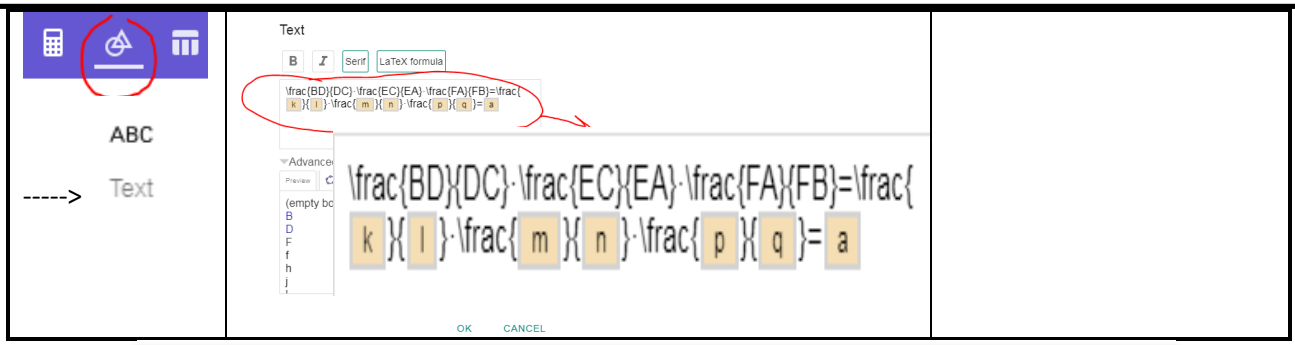
Isosceles triangle Construction with given base and givenline

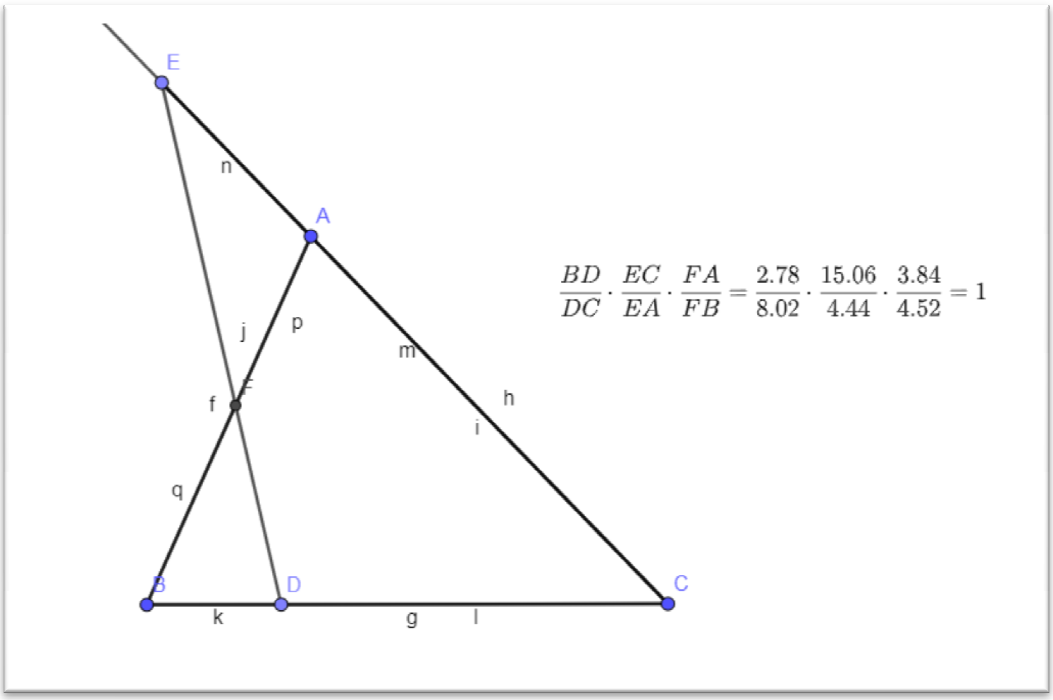
MENU	TOOL	PROCESS STEPS
	Segment	Draw a segment
	Line	Draw a linefromtwopoints C,D
	Midpoint or center	Clickthe segment AB to findthemidpoint E
	PerpedicularLine	Click E point and to the segment AB to draw
	Intersect	Clicktheperpedicularbisector and theCD line to findthepeak point of theisosceles triangle

EUROGEBRA WORKSHEET

MENELAOS THEOREM

MENU	TOOL	PROCESS STEPS
 Segment	Construct the triangle ABC	
 Point	Create a point D inside the segment BC	
 Ray	Create the ray CA	
 Point	Create a point E on the ray CA outside of the segment AC	
 Segment	Create the segment ED	
 Intersect	Create the intersect point F of ED and AB	
 Segment	Define the segments BD,DC,EC,EA,FA,FB (with this order)	
	Type: $\frac{k}{\ell} \cdot \frac{m}{n} \cdot \frac{p}{q}$ (that will give: a→1)	


ABC
----->
Text
Text
B
I
Serif
LaTeX formula
\frac{BD}{DC}·\frac{EC}{EA}·\frac{FA}{FB}=\frac{
k }{ l }·\frac{ m }{ n }·\frac{ p }{ q }= a
OK
CANCEL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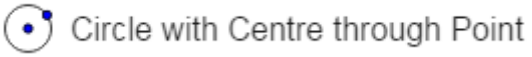
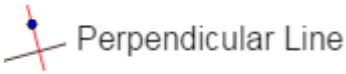
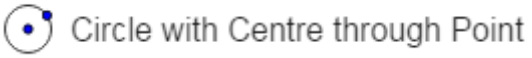
$\frac{BD}{DC}\cdot\frac{EC}{EA}\cdot\frac{FA}{FB}=\frac{2.78}{8.02}\cdot\frac{15.06}{4.44}\cdot\frac{3.84}{4.52}=1$

EUROGEBRA WORKSHEET

MENISCUS SQUARE

MENU	TOOL	PROCESS STEPS
		**Draw a circle of centre A and radius AB.**
		**Draw the segment AB.**
		**Click point A and AB segment to draw the perpendicular line g.**
		**Click the perpendicular line g and the circle A to find the intersection points C and D.**
		**Draw the segment DB.**
		**Click the D and then the B points to find the midpoint E.**
		**Click the E point to draw the circle through point B.**
		**Select a random point F in semicircle DB , outside A circle.**
		**Select a random point G in the convex arc DB, of A circle.**
		**Click the E point then B point and through point F , click the D point to shade the „e“ sector.**

		**Click the „e“ sector to messure its area. (This is the area of the semicircle EBFD)**
	 Polygon	**Click the D, A, B and again D points to shade the DAB triangle.**
		**Click the DAB to messure its area.**
	 Circular Sector	**Click the A point then B point and through point G , click the D point to shade the „k“ sector.**
		**Click the „k“ sector to messure its area. (This is the area of the circular sector ABGD)**

Task 1 : calculate the area of circular section EBGD, by subtracting the area of the triangle DAB from the circular sector ABGD.

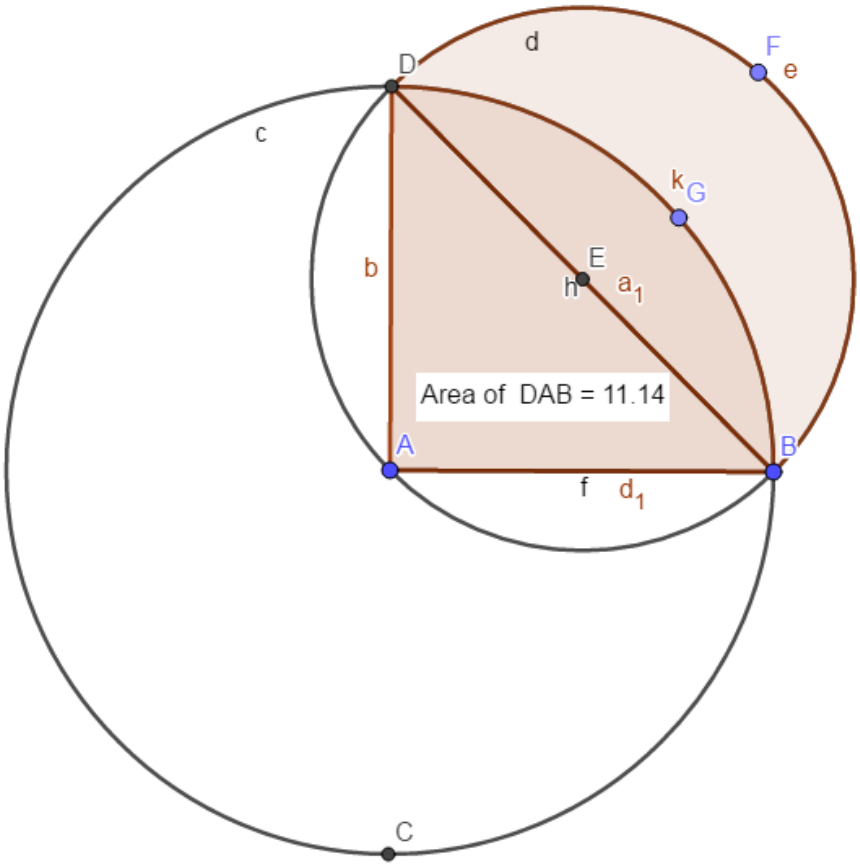
Task 2 : calculate the area of the meniscus DGBFD by subtracting the area of the circular section EBGD from the area of the semicircle EBFD.

Question : What do you notice about the area of the meniscus DGBFD and the area of the triangle DAB.

Explain your answer.

Area of DFBED - Area of DGBED = 17.5 - 6.36 = 11.14 = Area of DFBGD Meniscus = Area of DAB

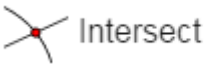
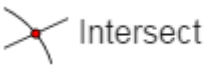
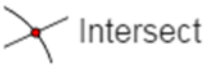
Area of ADGBA − Area of DAB = 17.5 − 11.14 = 6.36 = Area of DGBED

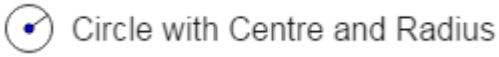
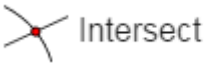
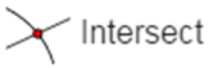


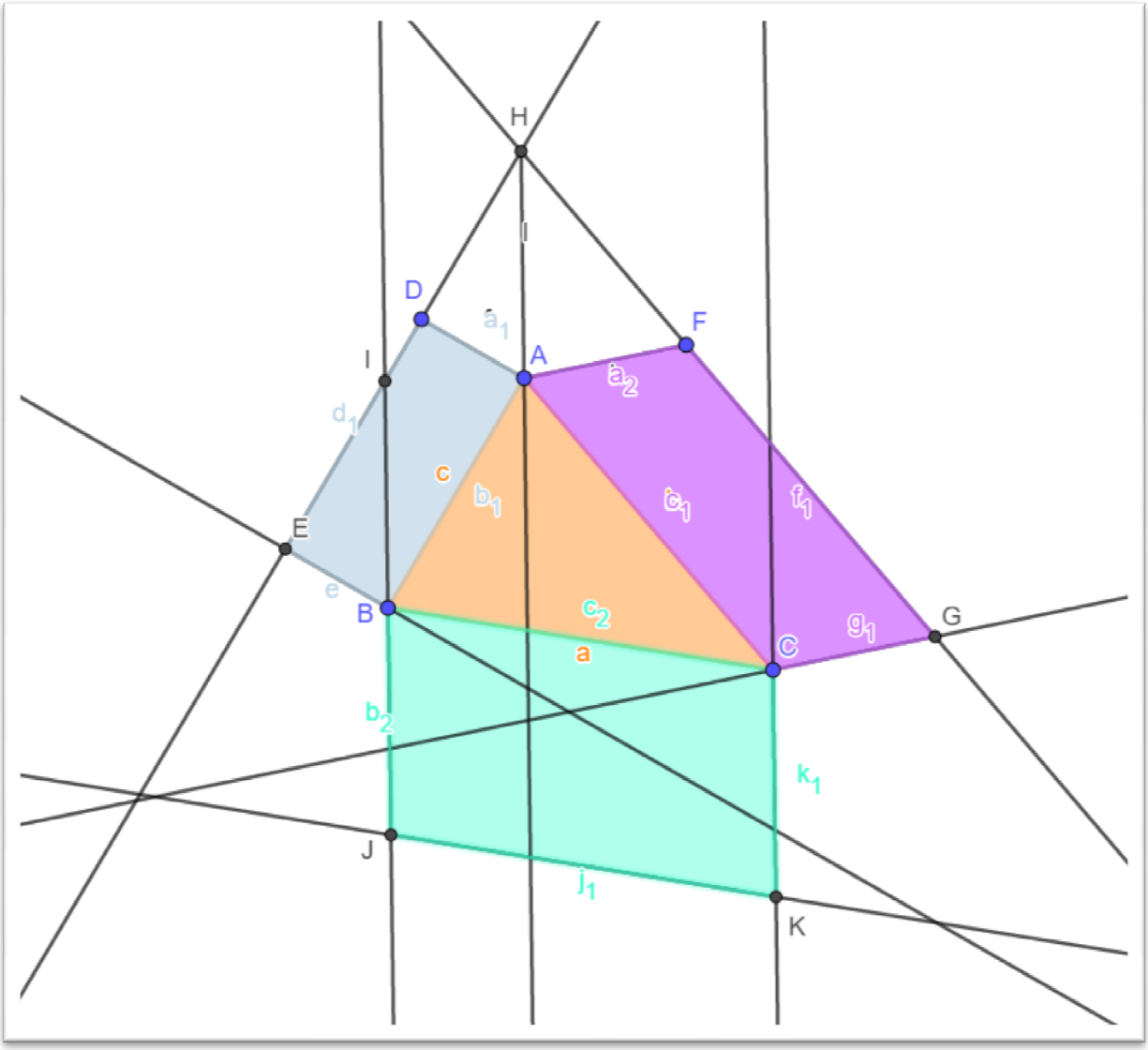
EUROGEBRA WORKSHEET

PAPPUS'S AREA THEOREM

Pappus's area theorem describes the relationship between the areas of three parallelograms attached to three sides of an arbitrary triangle. The theorem, which can also be thought of as a generalization of the Pythagorean theorem, is named after the Greek mathematician Pappus of Alexandria (4th century AD), who discovered it. (Wikipedia)

MENU	TOOL	PROCESS STEPS
		**Draw an arbitrary triangle ABC**
		**From point A draw a segment AD outside and to the left of the triangle**
		**From point D draw a parallelline to AB From point B draw a parallellineto AD**
		**Find the intersect point E for the 2 previous lines**
		**From point A draw a segment AF outside and to the right of the triangle**
		**From point F draw a parallel line to AC From point C draw a parallel line to AF**
		**Find the intersect point G for 2 the previous lines**
		**Find the intersect point H of the lines ED and GF**
		**Draw the ray HA**

		**From point B draw parallel to segment HA** **From point C draw parallel to segment HA**
		**Draw a cicle with center the point B and radius HA**
		**Find the inetrsect point J of the circle and the parallel line to AH**
Right click on circle and uncheck "show Object" to clear the circle		
		**Draw a parallel line from point J to segment BC**
		**Find the intersect point K between the parallel line from point J and the parallel line from the point C (Creating the parallelogram BJKC)**
Can you prove that $Area_{ABED} + Area_{AFGC} = Area_{BJKC}$?		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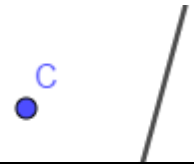
G
g_1
a
C
b_2
k_1
J
j_1
K

EUROGEBRA WORKSHEET

POINTS IN LANE

Reflect a point in a line

MENU	TOOL	PROCESS STEPS
		
	 Circle with Centre through Point	**Draw a circle with the center it at C and cutting the line at two points**
	 Intersect	**Define the points that formed with intersection of circle and line with the intersection tool (E and F points)**
	 Compasses	**Drawtwo circles with radius equal to length ECand center E and F**
	 Kesiştir	**Define the points that formed with intersection of circles that E and F-centered (G point)**

EUROGEBRA WORKSHEET

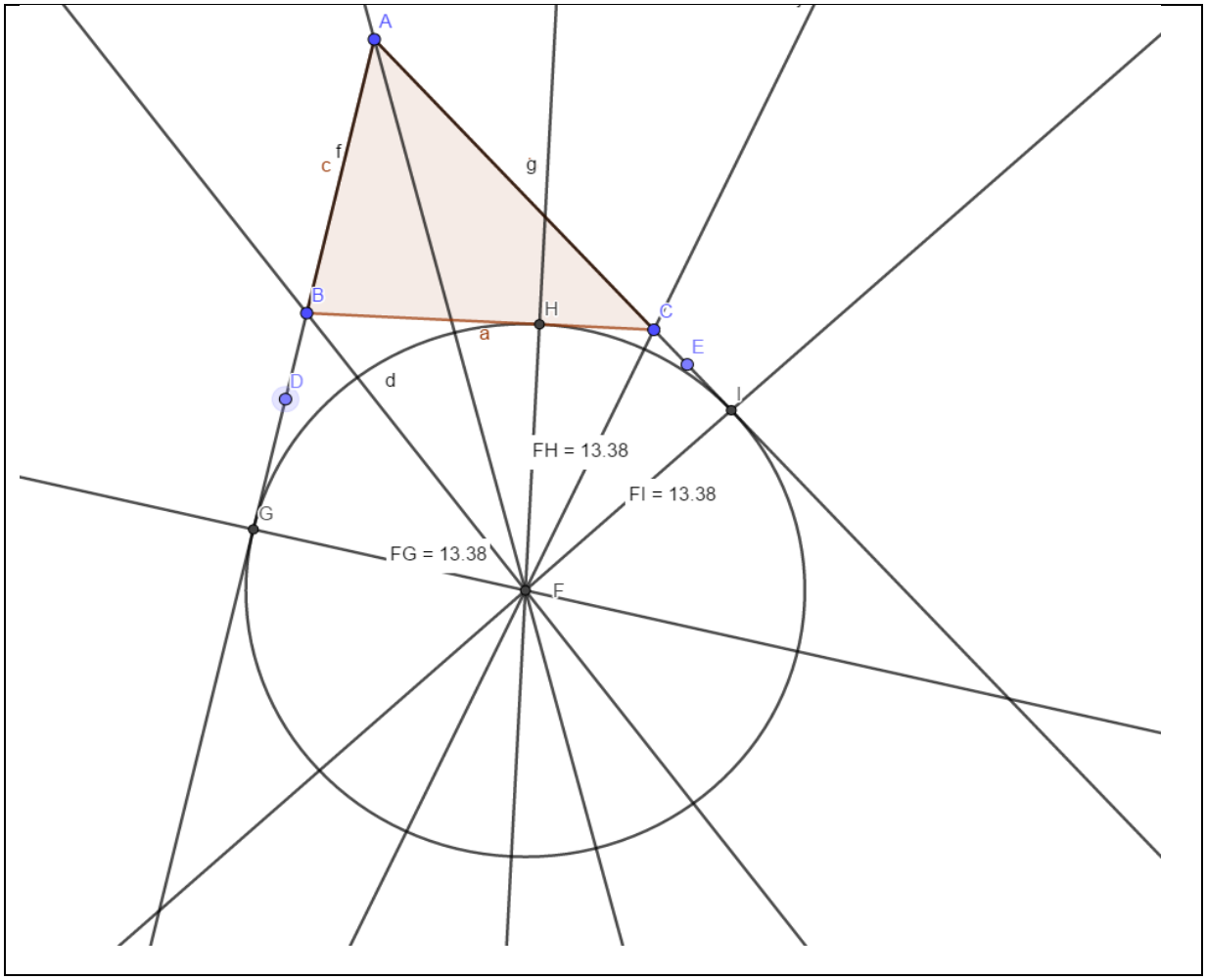
THE PRESCRIBED CIRCLES

MENU	TOOL	PROCESS STEPS
	 Polygon	**Constract a random triangle ABC**
	 Ray	**Construct the two rays from the point A by clicking points A ,B and A, C**
		**Click a random point D on the ray AB and a random point E on the ray AC both outside the triangle**
	 Angle Bisector	**Constract the BAC, DBC and BCE angles' bisectors**
	 Intersect	**Find the intersection point F of the three bisectors**
	 Perpendicular Line	**Draw the perpenticular lines from point F to the segments AB,BC,AC**
	 Intersect	**Find the intersection points between perpenticulars and the segments AB ,BC,AC (points G,H and I)**
	 Distance or Length	**Find the lehgths of the segments FG, FH and FI**

- **<u>Question1:</u>Can you explain why the lenghts are equal?**
- **<u>Question 2:</u>What can you say about the points G,H,I ?**

	 Circle through 3 Points	**Click the points G, H and I**

This is the (F,FG) prescribedcircle. Are there any others?



EUROGEBRA WORKSHEET

REGULAR POLYGON

A regular polygon and the sum of an internal point distances from its sides

MENU	TOOL	PROCESS STEPS
	 Regular Polygon	**Draw two points A and B and select 6 vertices to draw a regular hexagon.**
	 Segment	**Click the A, D points, the B, E points and the C, F points to draw the corresponding segments.**
	 Point	**Click to make the interior point G.**
	 Perpendicular Line	**Click the G point and the AB side to draw the perpedicular line to AB and DE. Repeate , clicking the G point and the BC side and once more, clicking the G point and CD side, so that the perpedicular lines intersect the sides at their internal points.**
	 Intersect	**Click, in succession, the perpendicular line to AB side and AB side, the perpendicular line to BC side and BC side, the perpendicular line to CD side and CD side, the perpendicular line to DE side and DE side, the perpendicular line to EF side and EF side, and the perpendicular line to FA side and FA side, to find the H, I, J, K, L and M points .**
	 Intersect	**Click AD and BE segments to find the intersection point N.**

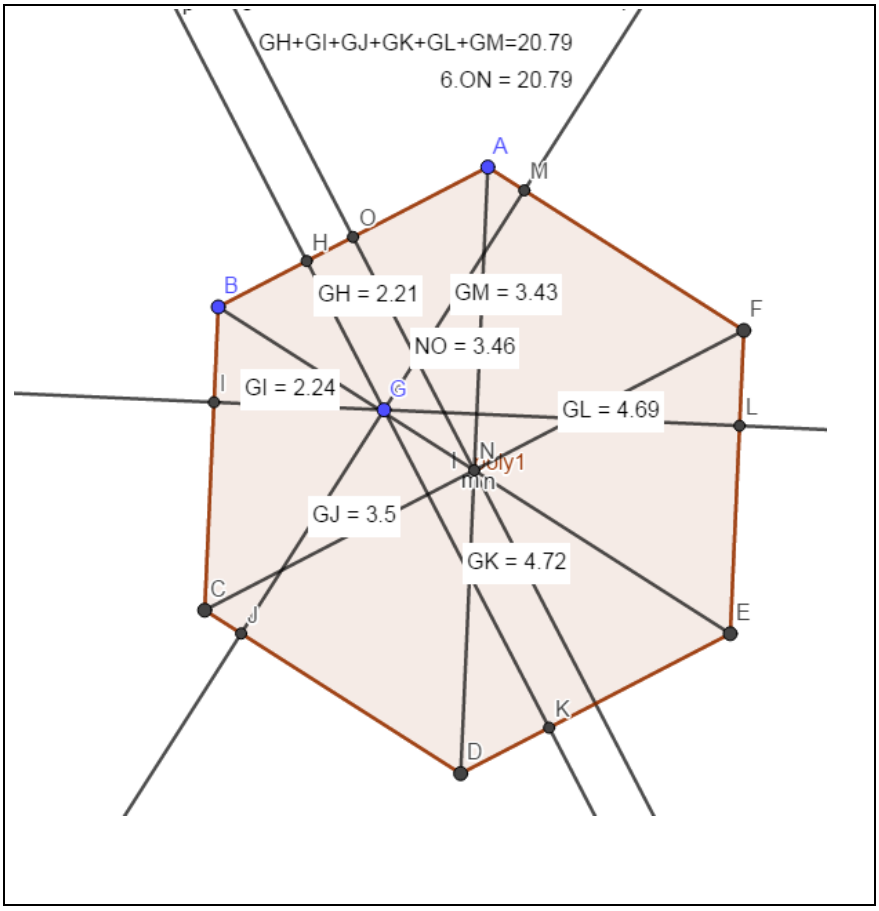
	 Perpendicular Line	**Click the N point and AB side to draw the perpendicular line.**
	 Intersect	**Click the last perpendicular line and AB side to find the O point.**
	 Distance or Length	**Click the G,H points, G,I points, G, J then G, K, then G, L then G, M and N, O points to messure the corresponding lengths**

Go to the „input“ section on the left and insert „GH + GI + GJ + GK + GL + GM =“ , click „enter“ and then „ 6 X NO =“

GH + GI + GJ + GK + GL + GM = a
and 6 x NO = b

Question : Prove the equality a = b.

Task : Make the same procedure on a 7 vertices regular polugon.



EUROGEBRA WORKSHEET

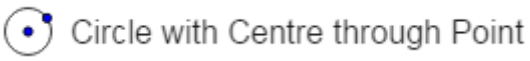
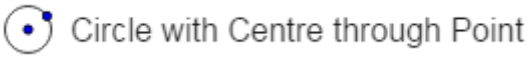
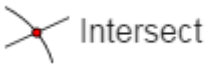
SQUARE CONSTRUCTION

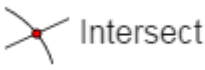
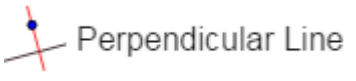
Square Construction with given side length

MENU	TOOL	PROCESS STEPS
	Segment withgivenlength	Draw a segment AB fromthe point A withgivenlenghte.g. 10
	Rotatearound point	1. Checkthe segment AB 2. Clickthe point A 3. Write 90^0degrees
	Rotatearound point	1. Checkthe segment A´B´ 2. Clickthe point B´ 3. Write 90^0degrees
	Rotatearound point	1. Checkthe segment A´´B´´ 2. Clickthe point A´´ 3. Write 90^0degrees

EUROGEBRA WORKSHEET

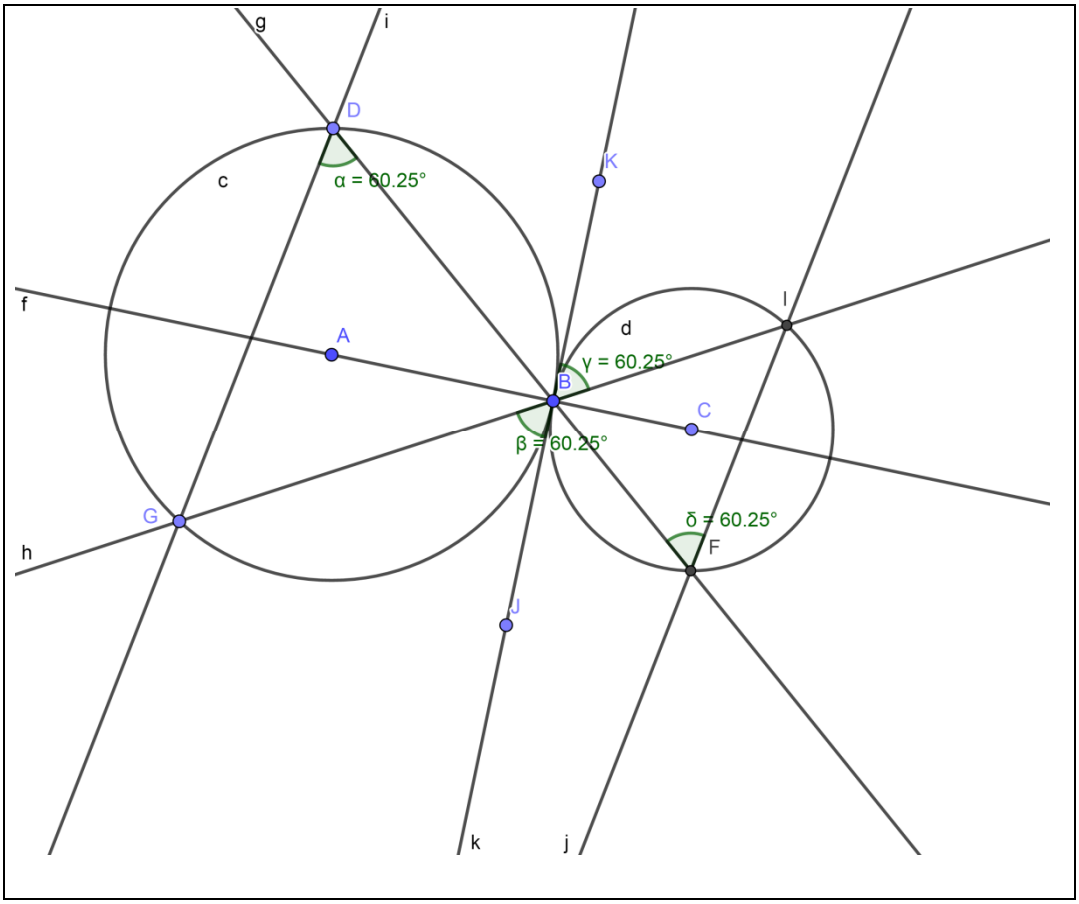
TANGENT CIRCLES

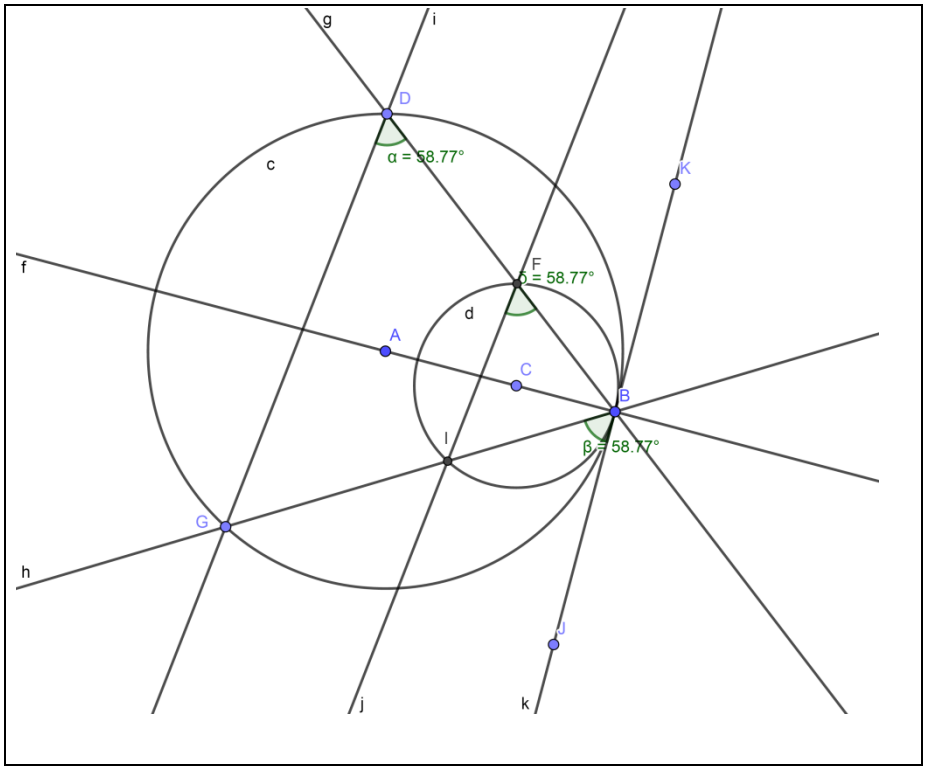
MENU	TOOL	PROCESS STEPS
		**Draw a circle with center point A through point B.**
		**Draw the AB line.**
		**Select a random point C on AB line. (out of circle A)**
		**Draw another circle with center point C through point B. (Let the second circle C have different radius from circle A)**
		**Select a random point D on A circle. (D, A, B non- consistently)**
		**Draw the DB line.**
		**Click the B circle and the DB line to find the intersection point F. (There will be another intersection point E identifed with B point. Click the grey dot on the left list, so only B point appears)**
		**Select another random point G on A circle. (G, A, B non- consistently)**
		**Draw the GB line.**

		**Click the B circle and the GB line to find the intersection point I. (Again there will be another intersection point H identifed with B point. Click the grey dot on the left list, so only B point appears)**
		**Draw the DG line.**
		**Draw the IF line.**
		**Click the B point and the AB line, to draw the perpendicular line.**

Question : What do you notice about the lines DG and IF. Explain your answer.

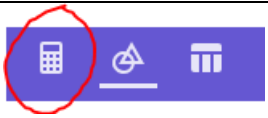
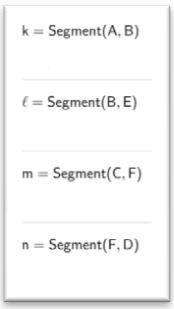
Task : Create your own procedure for the same problem if the circles are internally tangent.

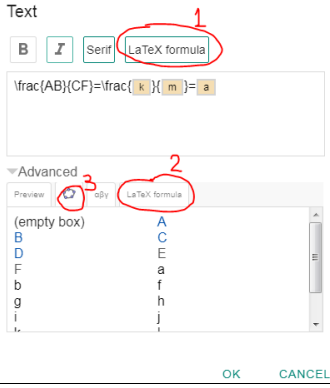
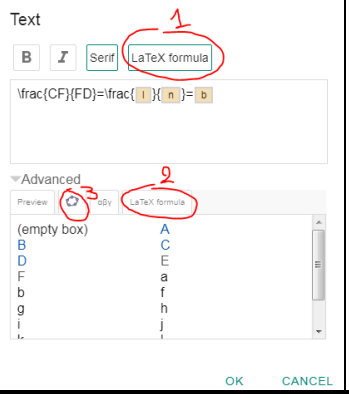

g
i
D
c
α = 60.25°
K
f
A
d
l
γ = 60.25°
B
C
β = 60.25°
G
δ = 60.25°
F
h
J
k
j

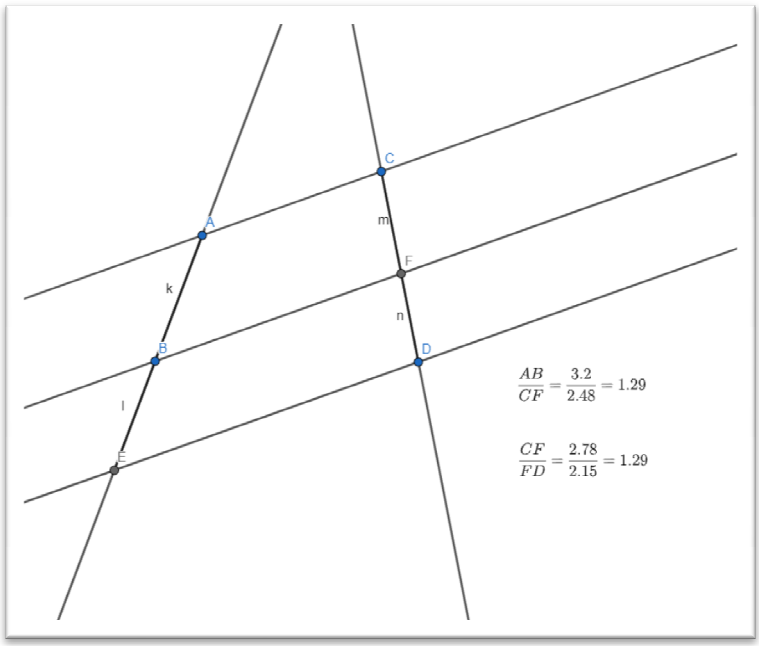

g
i
D
c
α = 58.77°
K
f
F
δ = 58.77°
d
A
C
B
l
β = 58.77°
G
h
J
j
k

EUROGEBRA WORKSHEET

THALES THEOREM

MENU	TOOL	PROCESS STEPS
 Line	Create a straight line of points A, B	
 Line	Similarly, a second line that is different from the first line, of points C, D	
 Line	From point A, make a straight line topoint C	
 Parallel Line	From Point B make a parallel to the straight AC	
 Parallel Line	From point D, make a parallel to the straight AC	
 Intersect	With this button, create all the points of intersection of the lines that are not defined (from left to right) so that that the points A, B, E are in the same line and also the points C, D F.	
 Segment	Define all straight parts AB, BE, CF and FD	
	Go to the Algebra section and type $\frac{k}{m}$ and then type $\frac{\ell}{n}$ (Then you'll have $a=\frac{k}{m}$ and $b=\frac{\ell}{n}$) 	

	Go back to Geometry and…	
	Create the adjacent text with the button 	
	Similarly, create the adjacent text with the button 	

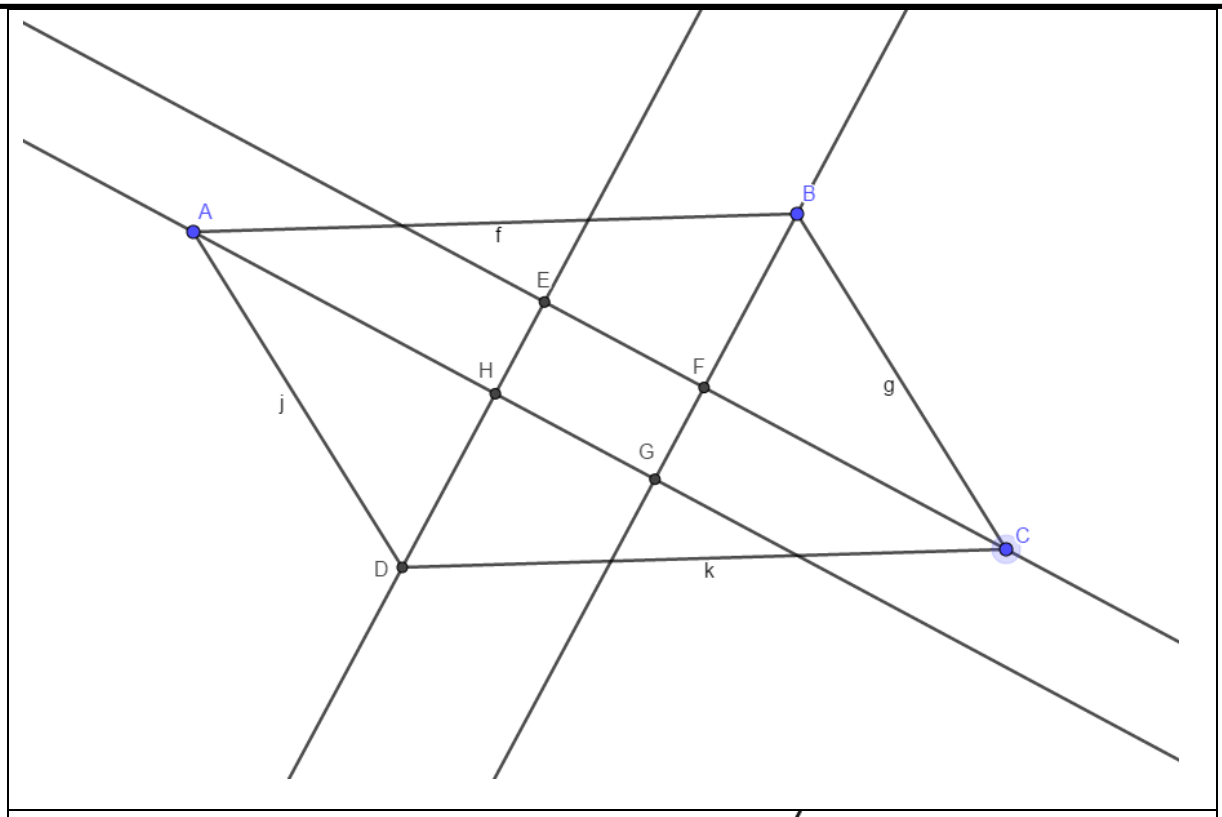


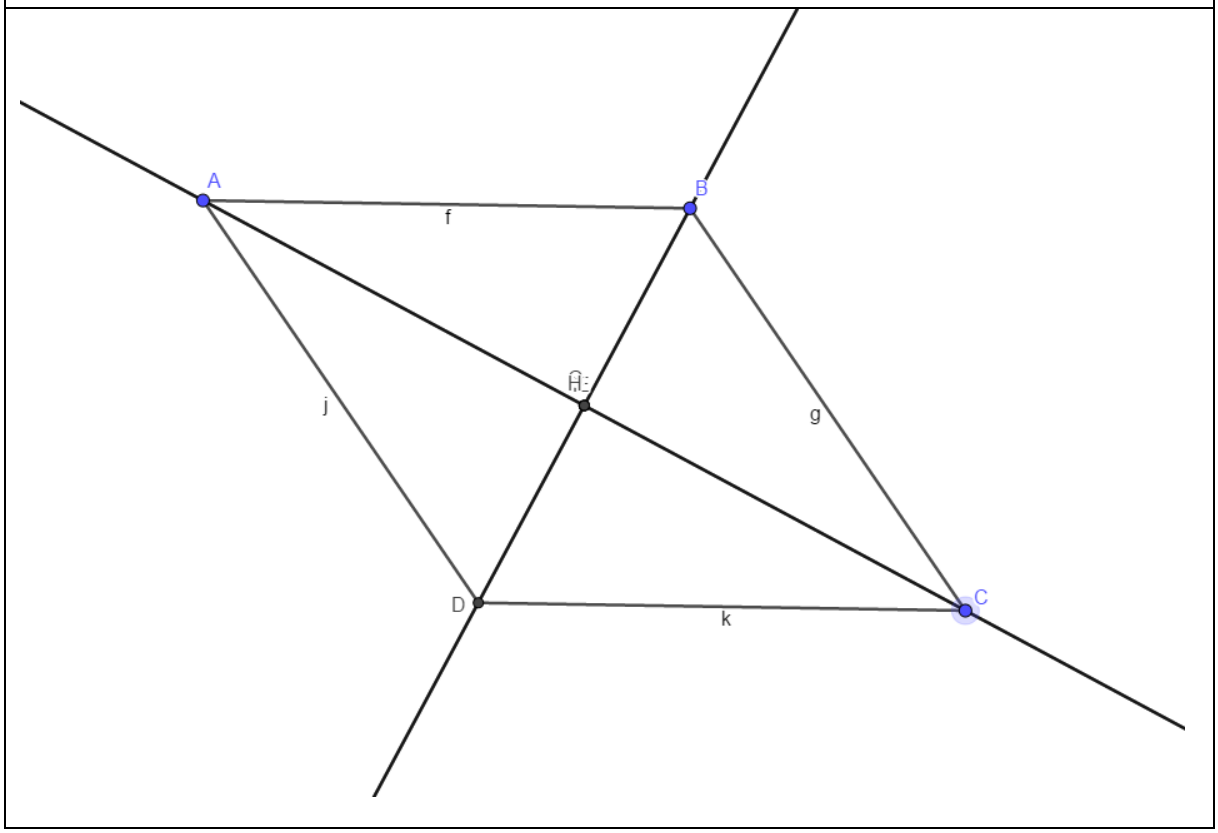
EUROGEBRA WORKSHEET

THE RECTANGULAR BISECTORS

MENU	TOOL	PROCESS STEPS
		**Draw three point A,b and C (non co-linear)**
		**Construct the segments AB and BC**
		**Click point B and segment AB and then point A and segment BC to construct the recragular**
		**Find the forth apex D of the rectagular**
Right click on each one of the parallels and uncheck the <<Show Object>> button		
		**Construct the segments AD and DC**
		**Construct the angle bisectors of all four angles of the rectagular**
		**Find the intersection points of the bisectors (Points E,F,G and H)**

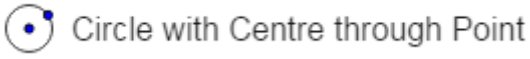
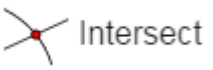
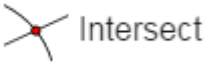
- **Question1: What kind of a shape is EFGH ? Explain...**
- **Question2: If all the bisectors meet to the same point , what kind of shape would be the ABCD rectagular?**


A
B
f
E
H
F
g
j
G
C
D
k

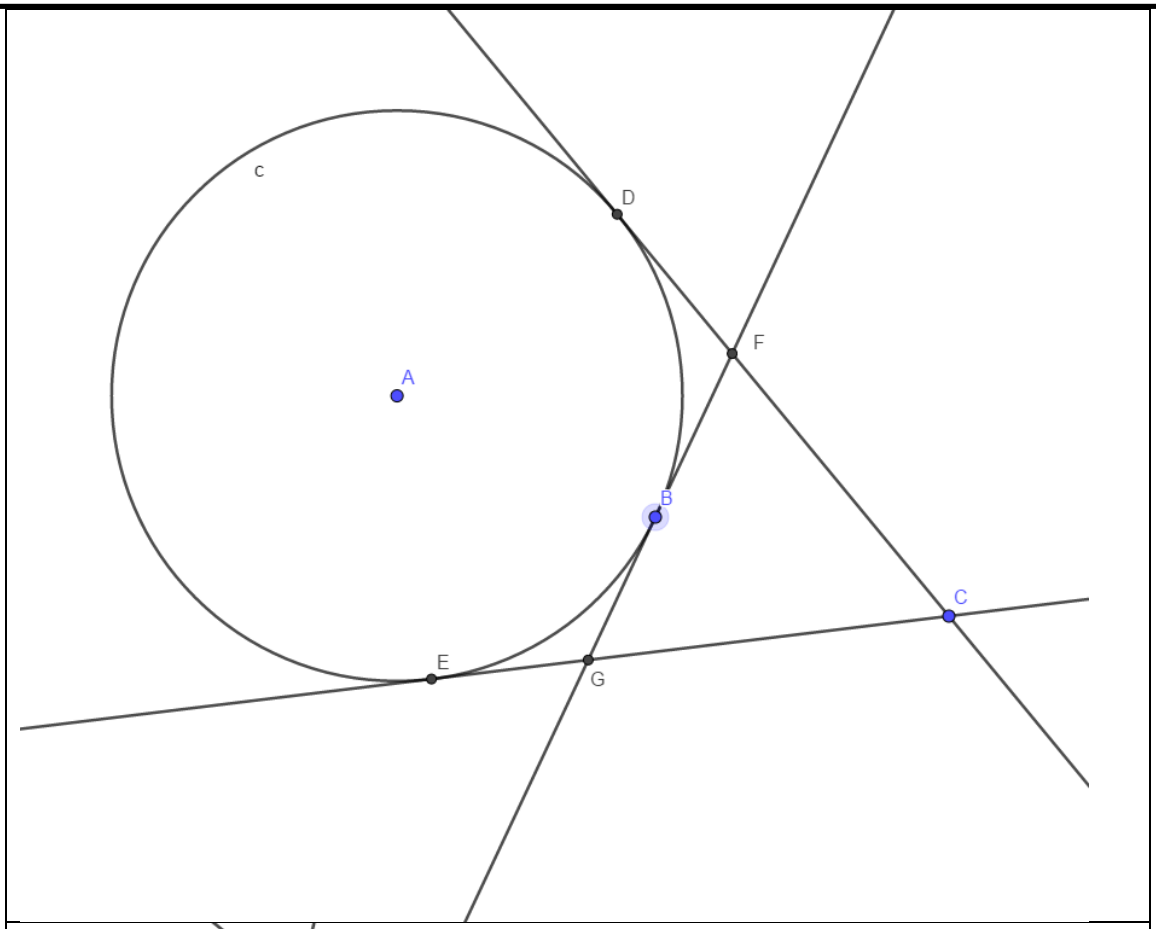

A
B
f
H
j
g
D
k
C

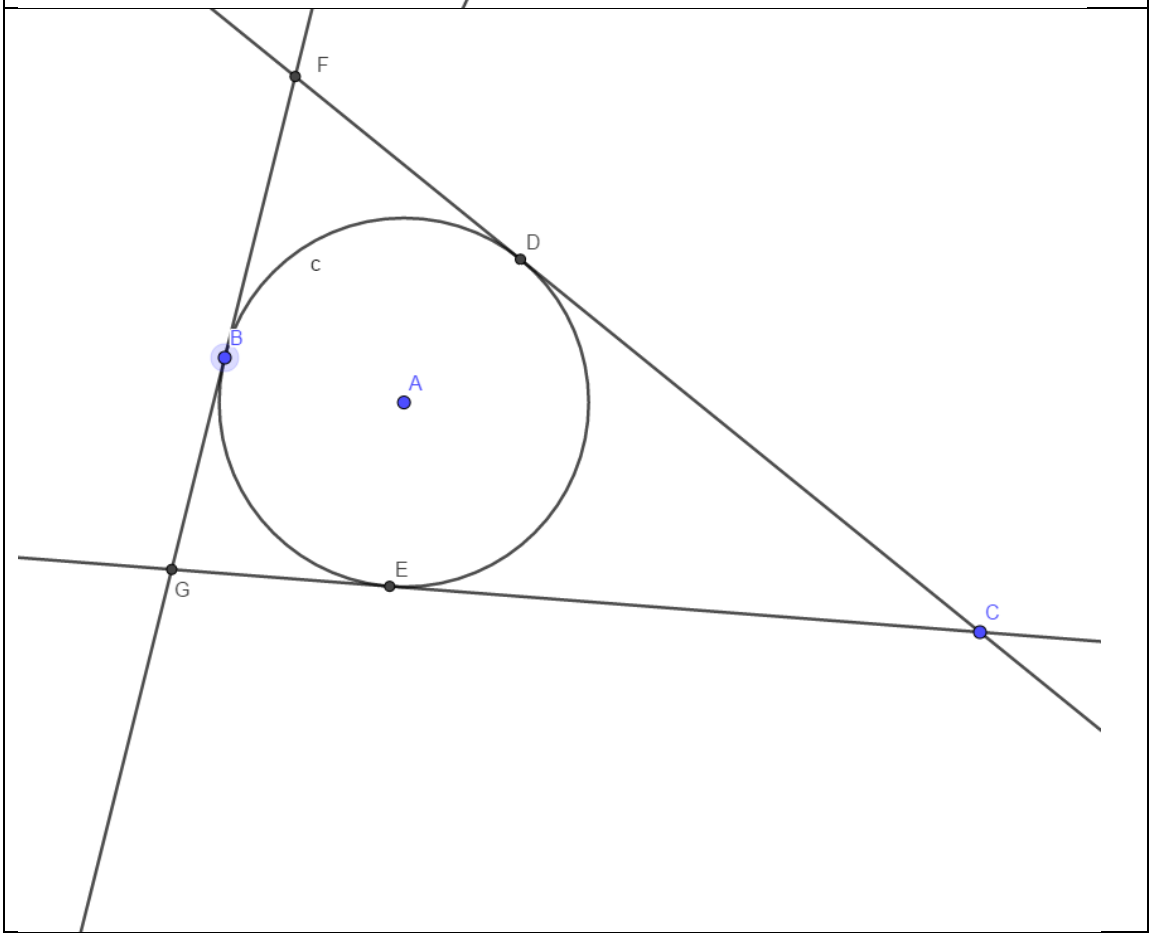
EUROGEBRA WORKSHEET

THE RELATION OF TRIANGLES PERIMETER AND TANGENTIAL SEGMENTS

MENU	TOOL	PROCESS STEPS
		**Click twice and draw point A and B**
		**Construct the circle with center A and trough point B**
		**Click a point C outside of the circle (point B must be between A and C)**
		**Constract the tangents from point C to the circle by clicking point C and the circle**
		**Constract the tangent from point B to the circle by clicking point B and the circle**
		**Find the intersection points between tangents and circle (points D and E)**
		**Find the intersection points between tangent from point B and tangential segments CD and CE**

- **<u>Question1:</u> Find the relation between triangles CFG perimeter and the lenght of the sengemt CD**
- **<u>Question 2:</u> If the point B belongs to the non-convex arc DE , find the relation of the triangles perimeter and the lenghts of the segments CD and FG**

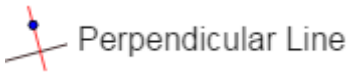
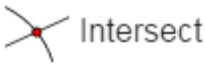
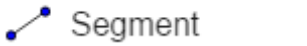
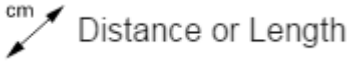

c
D
F
A
B
C
E
G

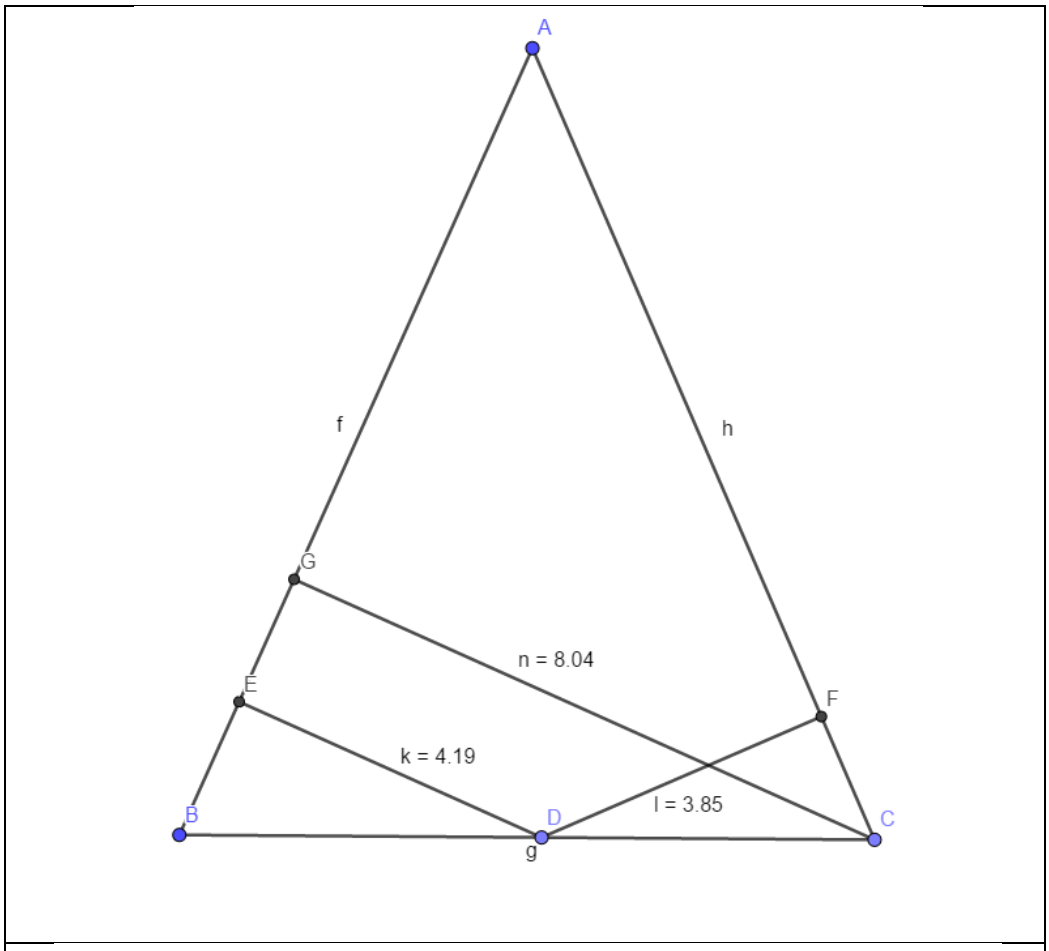

F
D
c
B
A
E
G
C

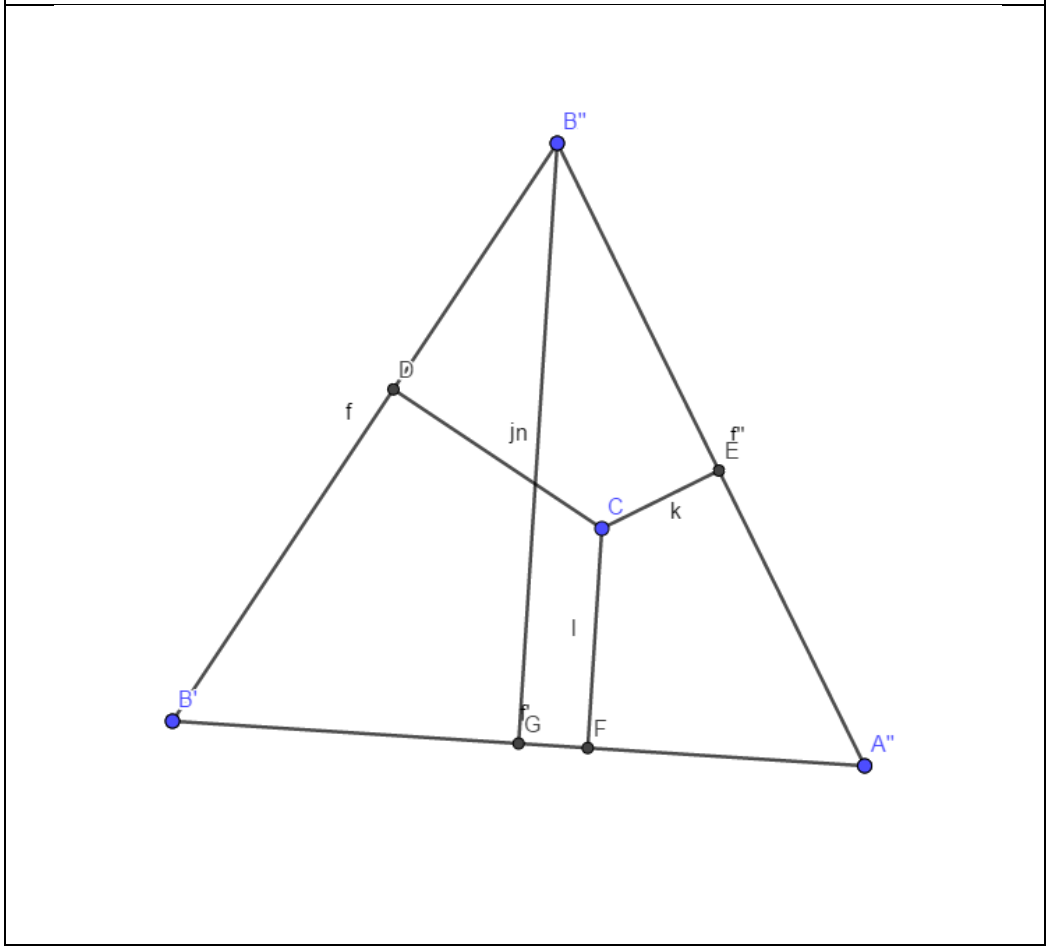
EUROGEBRA WORKSHEET

THE SUM OF THE DISTANCES OF A RANDOM POINT INSIDE OF AN ISOSCELES TRIANGLE IS CONSTANT

MENU	TOOL	PROCESS STEPS
	 Segment	**Draw the segment AB**
	 Circle with Centre through Point	**Draw the circle with center A and through point B**
	 Point	**Click the point C on the circle**
	 Segment	**Construct the segments BC and CA**
Right click on the circle and uncheck the <<Show Object>> button		
	 Point	**Draw a random point D on the segment BC**
	 Perpendicular Line	**Construct the perpendiculars from point D to the AB and AC segments**
	 Intersect	**Find the intersection points of perpendicular lines with the segments AB and AC (Points E and F)**
Right click on the perpendiculars and uncheck the <<Show Object>> button		
	 Segment	**Draw the segments DE and DF**

		**Construct the perpendicularfrom point C to the AB segment**
		**Find the intersection point of perpendicular line with the segments AB (Point G)**
Right click on the perpendicular and uncheck the <<Show Object>> button		
		**Draw the segment CG**
		**Measure the lenghts of the segments DE ,Df and CG**
<u>Question1:</u>Can you find the relation between the lengths of segments? **<u>Question2:</u> What happens if the initial triangle is equilateraland D is an inner point of the triangle ?**		


A
f
h
G
n = 8.04
E
F
k = 4.19
l = 3.85
B
D
C
g


B"
D
f
jn
f"
E
C
k
l
B'
G
F
A"