EUROGEBRA WORKSHEETS

3D Geometry

Quadratic (In)Equations

EUROGEBRA WORKSHEETS

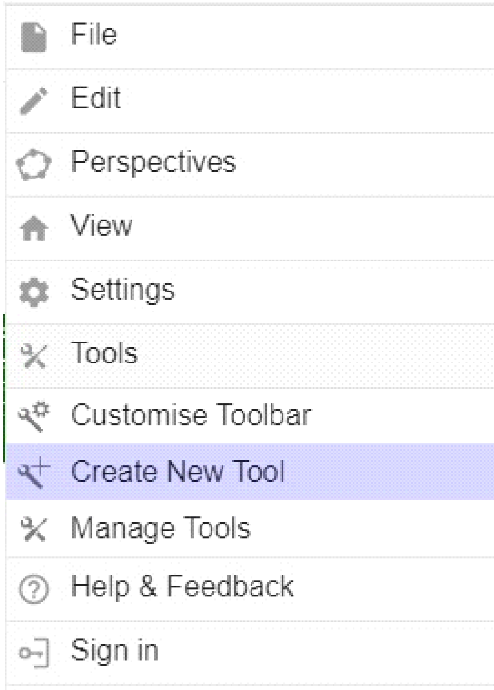
Introduction:

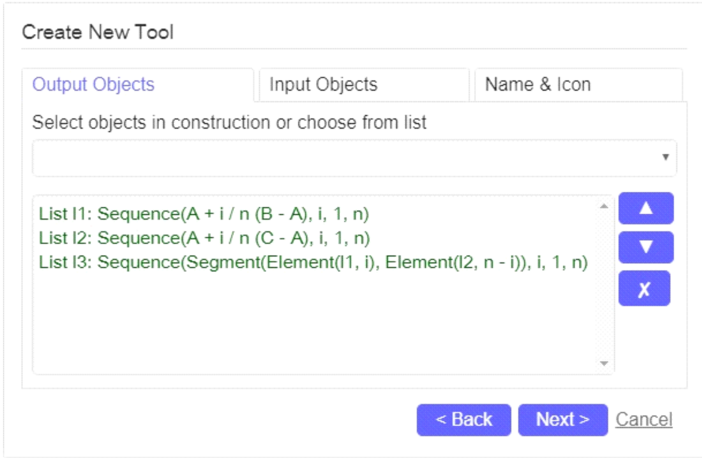
These worksheets were created within the Erasmus + project, Eurogebra.
Worksheets are in the field of mathematics and use the Geogebra program for individual mathematical tasks. The purpose is to use the program when teaching and explaining problems in mathematics and thus to approach the issue more clearly.
Worksheets are in the form of specific instructions and tools that will guide us to solve various tasks. In this way, students will get closer to a better understanding and modification of the given examples. Individual groups of worksheets can be combined with each other and create new subgroups according to the issues discussed. Some examples are followed by the solution of examples and free sheets for student notes.
Worksheets respect pedagogical documents related to the subject of mathematics. When working with worksheets, it is necessary to cooperate with teachers and coordinate their work.
In terms of content, we focused on geometric examples, where you can effectively solve problems and modify them in various ways. By formulating the tasks, we lead the students to understand the assigned tasks and to solve the tasks according to the individual steps in the worksheets.

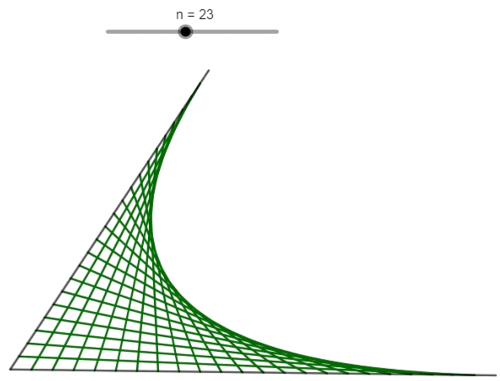
EUROGEBRA WORKSHEET

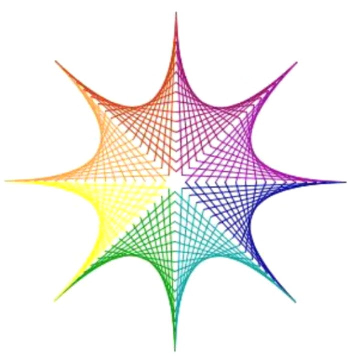
BEZIER CURVE

MENU	TOOL	PROCESS STEPS
		In *Settings* set *Labelling* to *All New Objects*
	Segment	Create a segment *AB (a)*
	Segment	Create a segment *AC (b)*
	Slider	Create a slider *n* MIN: 0, MAX 50, Increment: 1
		In the *input bar* type: *Sequence(A+i/n * (B-A),i,1,n)* This will create a list of *n* points on the AB segment. The distance between each point is exactly 1/n * length of *a.*
		In the *input bar* type: *Sequence(A+i/n * (C-A),i,1,n)* This will create a list of *n* points on the AC segment. The distance between each point is exactly 1/n * length of *b.*
		Hide both lists of points
		Create a list of segments: Sequence(Segment(Element(l1,i),Element(l2,n-i)),i,1,n) These segments will connect i-th element of l1 list

		with (n-i)-th element of l2 list
		You can move points A, B and C to change the shape of the Bezier Curve. Use the slider to change the number of segments.
		Hide the lebels of segments and points.
		Create a new tool called *Bézier Curve* Choose all the *lists* as *Output objects.* 

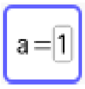
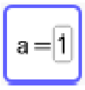
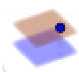
		
		Use the new tool to create more shapes





EUROGEBRA WORKSHEET

CARTESIAN EQUATION OF A PLANE

MENU	TOOL	PROCESS STEPS
	Create four sliders	**Create four sliders a, b, c and d . The slider will default to a range of –5 to 5.**
		Type ax+by+c=d in the algebra view window
	Create three sliders	**Create three sliders d ,f and g . The slider will default to a range of –5 to 5. Do not use e as Geogebra thinks it is the Euler number**
	Plot a 3D coordinate point in the algebra view	**Type (d,f,g) in the algebra view window**
	Click on the parallel plane icon on the geometry menu	**Select point D and the plane(A,B,C)**

„solution image"

Questions:

1. Write down the cartesian equation of a plane tha passes through the origin. Generalise your observation. Will be of the form n1x+n2y+n3z=0
2. Write down the cartesian equation of a plane that is parallel to plane p. Generalise your observation. Will have the same n1,n2,n3 values but will not equal to the same number.

EUROGEBRA WORKSHEET

MIN AND MAX VALUES OF A QUADRATIC FUNCTION IN A GIVEN INTERVAL

MENU	TOOL	PROCESS STEPS
		For this task it is recommended to use both *Graphics* and *Graphics 2* views. One for the sliders, dynamic texts, check boxes etc. and the other for the graph.
a=2	a=2 Slider	**Insert sliders *a*,*b* and *c*:** **MIN: -5, MAX: 5, increment 0.1**
	Input...	**In the input bar type in:** **f(x) = ax^2+bx+c**
	Input...	**In the input bar type in:** **g(x)=Polynomial(f)**
a=2	ABC Text	**Insert text: *Set the extremities of the closed interval:***
a=2	a=2 Slider	**Insert slider *d*:** **MIN: -10, MAX: 10, increment 0.1**
a=2	a=2 Slider	**Insert slider *e*:** **MIN: *d*+0.1, MAX: 10, krok 0.1**
	Input...	**In the input bar type in:** **eq1: x=*d***
	Input...	**In the input bar type in:** **eq2: x=*e***

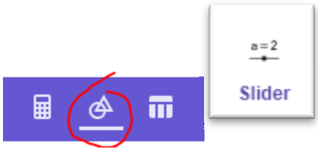
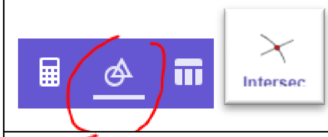
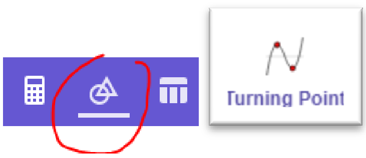
	Input…	**In the input bar type in:** **A = Intersect(g, eq1)**
	Input…	**In the input bar type in:** **B = Intersect(g, eq2)**
	Input…	**In the input bar type in:** **C = MIN(g,d,e)**
	Input…	**In the input bar type in:** **D = MAX(g,d,e)**
	Input…	**In the input bar type in:** $p = -\frac{b}{2a}$
	Input…	**In the input bar type in:** **q = f(p)**
	Input…	**In the input bar type in:** **k = f(d)**
	Input…	**In the input bar type in:** **m = f(e)**
	Input…	**In the input bar type in:** **W = (p,q)**
	Input…	**In the input bar type in:** **y_max = If(d≤p≤e, Max(k, Max(m,q)), Max(k,m))**
	Input…	**In the input bar type in:** **y_min = If (d≤p≤e, Min(k, Min(m,q)), Min(k,m))**
a = 2	ABC Text	**Insert dynamic text:** f(d)= k f(e)= m

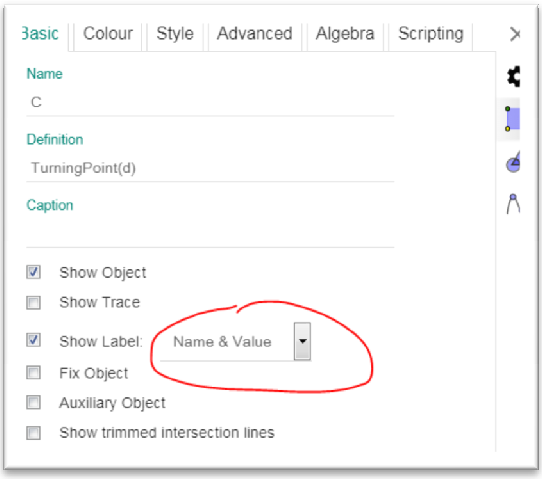
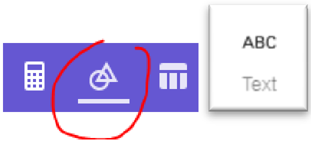
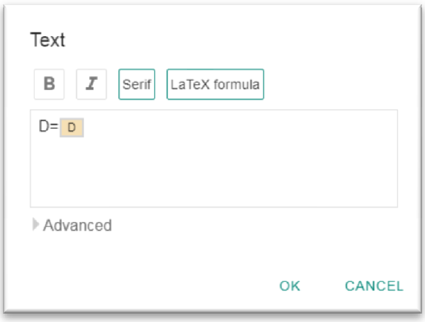
a = 2	Check Box	**Insert a check box: *Function value at the ends of the interval:*, which will show/hide the dynamic text from the previous step**
a = 2	ABC Text	**Insert dynamic text:** W=(p ; q) Set a condition to show this object: $i \wedge a \neq 0$
a = 2	Check Box	**Insert a check box: *Vertex of a parabola:*, which will show/hide the dynamic text from the previous step**
a = 2	ABC Text	**Insert dynamic text:** y_{min}= y_{min} y_{max}= y_{max}
a = 2	Check Box	**Insert a check box: *MIN and MAX values in given interval:*, which will show/hide the dynamic text from the previous step**
a = 2	ABC Text	**Insert text: *no vertex*** Set a condition to show this object: $i \wedge a \stackrel{?}{=} 0$
a = 2	ABC Text	**Insert dynamic text:** Examined interval: d $\leq x \leq$ e
a = 2	ABC Text	**Insert dynamic text:** f(x)= g
a = 2	ABC Text	**Insert dynamic text:** $x \in$ < d ; e >
a = 2	ABC Text	**Insert text: *Set the parameters of the quadratic function:***
	Input...	**In the input bar type in:** **z(x) = If (d≤x≤e, g(x))** Set a condition to show this object: j

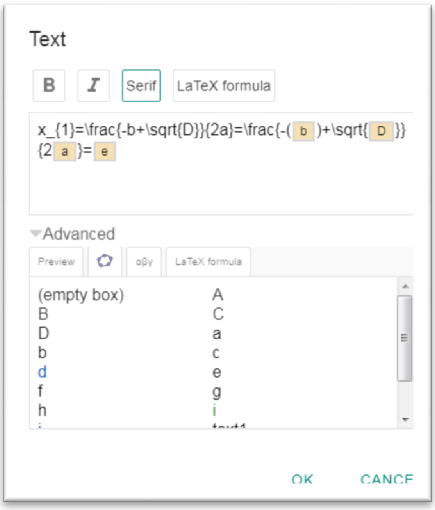

Ustaw współczynniki funkcji kwadratowej:
a = 0.5
b = -1.7
c = 0.3
Podaj krańce przedziału domkniętego:
d = 2.2
e = 5.4
Rozpatrywany przedział: 2.2 ≤ x ≤ 5.4
Parabola ma wierzchołek w punkcie:
W=(1.7;-1.15)
Wartość funkcji na krańcach przedziału:
f(2.2)=-1.02 f(5.4)=5.7
wartości min i max w podanym przedziale wynoszą:
$y_{min} = -1.02$ $y_{max} = 5.7$
$f(x) = 0.5\,x^2 - 1.7\,x + 0.3$
$x \in < 2.2; 5.4 >$
MAX
MIN
W

EUROGEBRA WORKSHEET

QUADRATIC EQUATION

PROCES STEPS	
	Create 3 sliders a,b,c
	$d(x) = a\,x^2 + b\,x + c$
	$b^2 - 4\,a\,c$ and then name it D
	Intersect(d, xAxis)
	$e = \frac{-b + \sqrt{D}}{2\,a}$ $f = \frac{-b - \sqrt{D}}{2\,a}$ $g = e + f$ $h = e\,f$
	Click on parabola d and the point C will appear. Then right click on the point C and: Point C(1.9, -0.81) Duplicate Fix Object Show Trace Settings

<table>
<tr>
<td></td>
<td>
</td>
</tr>
<tr>
<td></td>
<td>x= (-b)/(2a)
y= (-D)/(4a)</td>
</tr>
<tr>
<td>
</td>
<td>


</td>
</tr>
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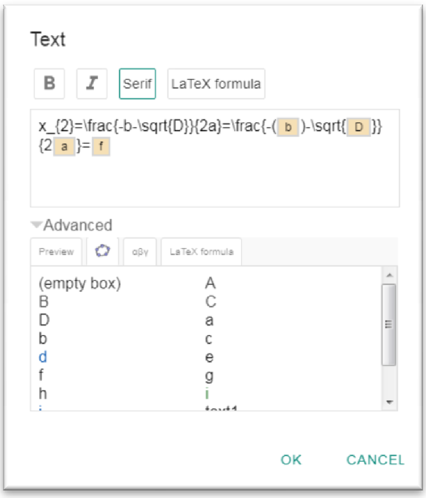
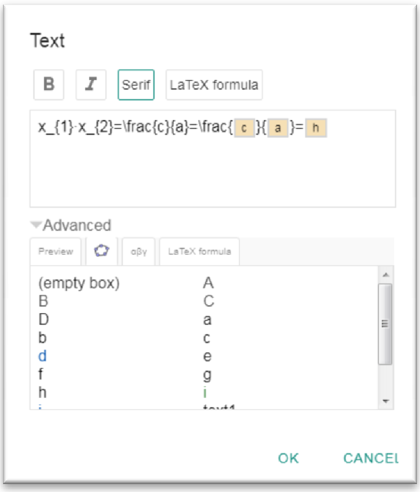
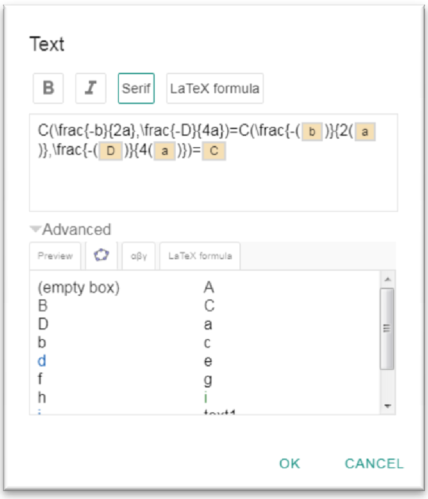

Text
B
I
Serif
LaTeX formula
x_{2}=\frac{-b-\sqrt{D}}{2a}=\frac{-(b)-\sqrt{ D }}{2 a }= f
Advanced
(empty box)
A
B
C
D
a
b
c
d
e
f
g
h
i
OK
CANCEL


Text
B
I
Serif
LaTeX formula
x_{1}+x_{2}=\frac{-(b)}{a}=\frac{-(b)}{ a }= g
Advanced
(empty box)
A
B
C
D
a
b
c
d
e
f
g
h
i
OK
CANCEL


Text
B
I
Serif
LaTeX formula
x_{1}·x_{2}=\frac{c}{a}=\frac{ c }{ a }= h
Advanced
(empty box)
A
B
C
D
a
b
c
d
e
f
g
h
i
OK
CANCEL

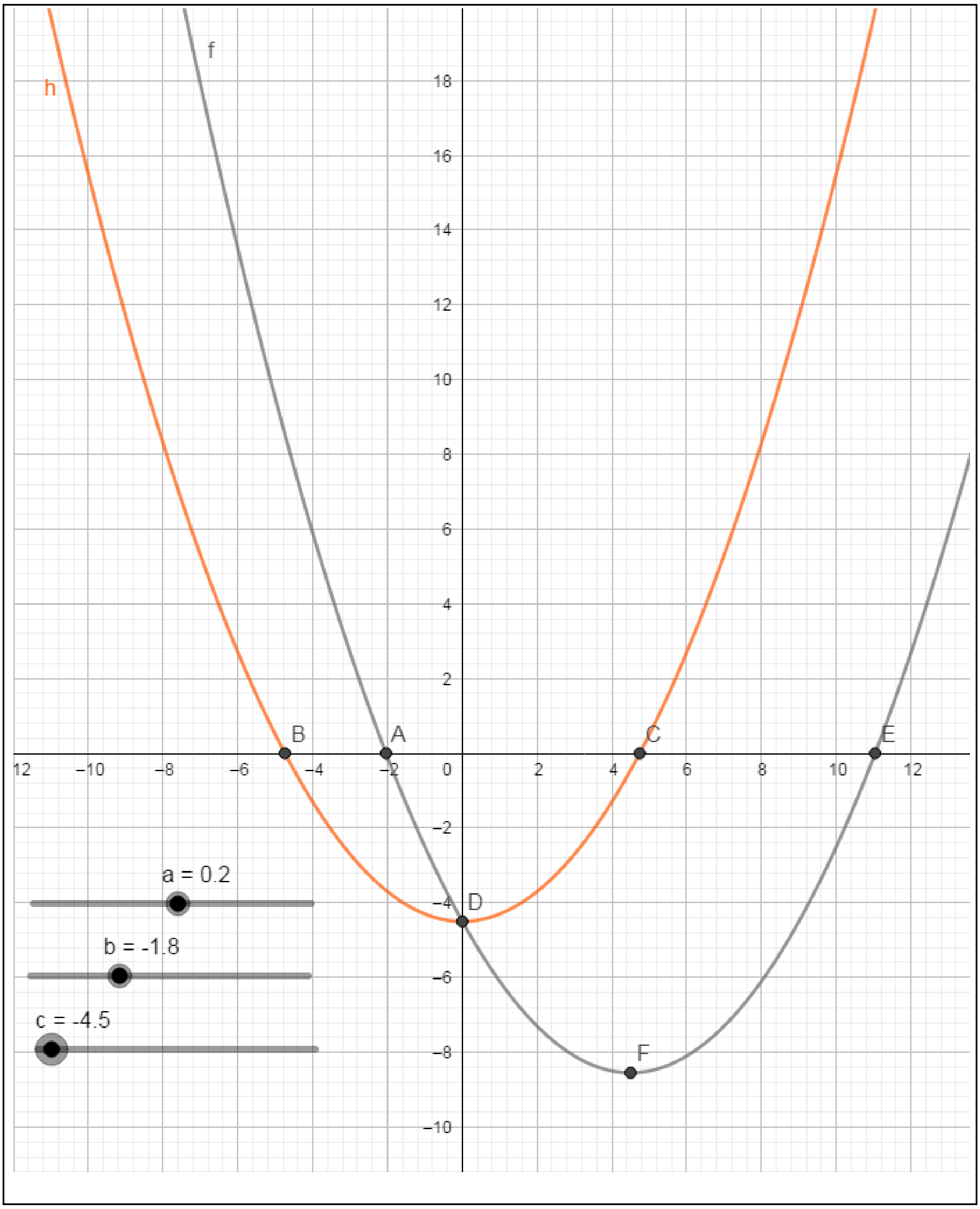
	Text B I Serif LaTeX formula C(\frac{-b}{2a},\frac{-D}{4a})=C(\frac{-(b)}{2(a)},\frac{-(D)}{4(a)})= C Advanced Preview αβγ LaTeX formula (empty box) A B C D a b c d e f g h i OK CANCEL

EUROGEBRA WORKSHEET

QUADRATIC EQUATIONS

MENU	TOOL	PROCESS STEPS
Write in the input cell the function „ x^2 ” to create the curve f. The curves´ name is parabola. “the $y = x^2$ parabola”		
a = 2	a = 2 Slider	Cilck on the geogebra board to define a slider „a“ , set min = - 5 and max = 5.
Write in the input cell the function „ $a{\cdot}x^2$ ” to create the curve g.		
Left click on the „ a“ slider‘s dot and move it , to see the relation between the two curves.		
A	Intersect	Click on the g curve and the x΄x axis to see the intersection point A. The A point is the extreme point of the parable.
1st task : What is the solution of the equation $ax^2 = 0$, $a \neq 0$. (show the solution in the graph)		
a = 2	a = 2 Slider	Cilck on the geogebra board to define a slider „c“ , set min = - 5 and max = 5.
Write in the input cell the function „ $a{\cdot}x^2 + c$” to create the curve h.		
Left click on the „ c“ slider‘s dot and move it , to see the relation between the g and h curves.		

<table>
<tr><td></td><td>Intersect</td><td>Click on the h curve and the x′x axis to see the intersection points B and C.
Then click again the h parable and the y′y axis to create the D point which is the extreme point of the h parable.</td></tr>
<tr><td colspan="3">Notice that , if a > 0, then the extreme point A and D are minimum,
if a < 0, then the extreme point A and D are maximum.</td></tr>
<tr><td colspan="3">2nd task : What is the solution of the equation $ax^2 + c = 0$, $a \neq 0$.
(show the solution in the graph)</td></tr>
<tr><td>a = 2</td><td>a = 2 Slider</td><td>Cilck on the geogebra board to define a slider „b“ , set min = - 5 and max = 5.</td></tr>
<tr><td colspan="3">You can delete the first two parables and
write in the input cell the function „ $a{\cdot}x^2 + bx + c$ ” to create the new curve f.</td></tr>
<tr><td></td><td>Intersect</td><td>Click on the f parable and the x′x axis to see the intersection point A and E.</td></tr>
<tr><td colspan="3">Write in the input cell the coordinates „(-b/2a, f(-b/2a))“ and then „enter“.
The F point appears on the f parable and this point is, always, the extreme point of any parable f.</td></tr>
<tr><td colspan="3">3rd task : What is the solution of the equation $ax^2 + bx + c = 0$, $a \neq 0$.
(show the solution in the graph)</td></tr>
</table>


h
f
A
B
C
D
E
F
a = 0.2
b = -1.8
c = -4.5

EUROGEBRA WORKSHEET

SHOW THE SOLUTIONS TO A QUADRATIC INEQUALITY

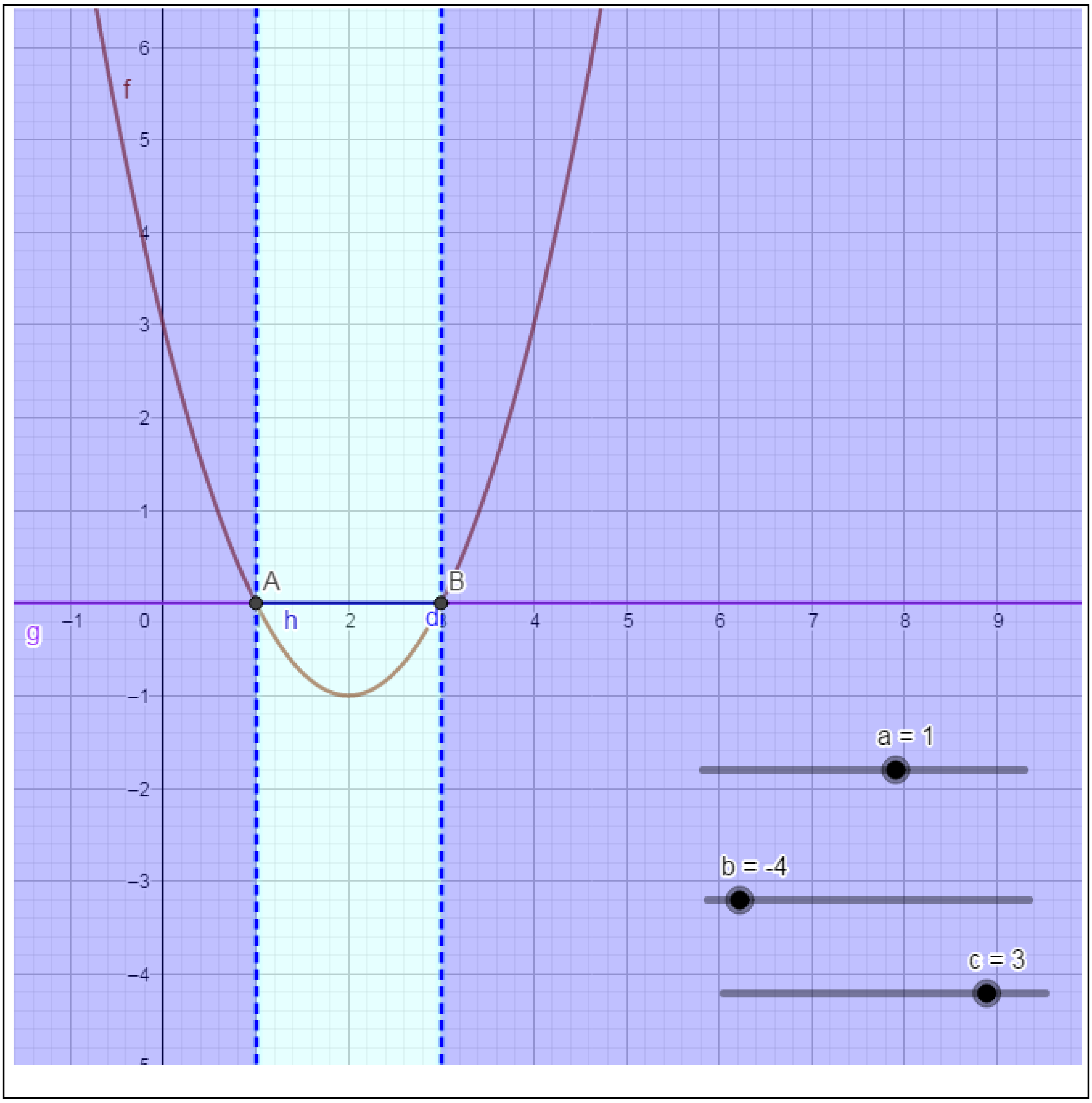
MENU	TOOL	PROCESS STEPS
a = 2	a = 2 Slider	**Create a slider for the variable "a" between -5 and 5.**
a = 2	a = 2 Slider	**Create a slider for the variable "b" between -5 and 5.**
+	Input...	**Input the equation "y = (x – a)(x – b)".**
+	Input...	**Input the inequality"0> (x – a)(x – b)".**
+	Input...	**Input the inequality "0 < (x – a)(x – b)".**

EUROGEBRA WORKSHEET

QUADRATIC INEQUALITIES

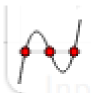
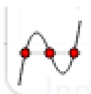
<table>
<tr><th>MENU</th><th>TOOL</th><th>PROCESS STEPS</th></tr>
<tr><td>a = 2</td><td>a = 2 Slider</td><td>Cilck on the geogebra board to define a slider „a“ , set min = - 5 and max = 5.</td></tr>
<tr><td>a = 2</td><td>a = 2 Slider</td><td>Cilck on the geogebra board to define a slider „b“ , set min = - 5 and max = 5.</td></tr>
<tr><td>a = 2</td><td>a = 2 Slider</td><td>Cilck on the geogebra board to define a slider „c“ , set min = - 5 and max = 5.</td></tr>
<tr><td colspan="3">Write in the input cell the function „ $a \cdot x^2 + bx + c$ ” to create the parabola f.</td></tr>
<tr><td colspan="3">Move the sliders so that the parabola f intersects the x Axis</td></tr>
<tr><td>A</td><td>Intersect</td><td>Click on the f parabola and the x΄x axis to see the intersection points A and B.</td></tr>
<tr><td colspan="3">Write in the input cell, the inequality „ f(x) > 0 ” .
Then “enter” and the set “d” appears on the left column.</td></tr>
<tr><td colspan="3">Write in the input cell „ if (d, 0) ” to color the x Axis section that solves the inequality .</td></tr>
<tr><td colspan="3">1st task : What is the solution of the inequality $x^2 - 4x + 3 > 0$.
(show the solution in the graph)</td></tr>
</table>

Write in the input cell, the inequality „ f(x) < 0 ” . **Then “enter” and the set “e” appears on the left column.**
Write in the input cell „ if (e, 0) ” to color the x Axis section that solves the inequality . **(use different color, from the settings, for the “e” set)**
2nd task : What is the solution of the inequality $-x^2 + 5x - 6 < 0$. (show the solution in the graph)
3rd task : What is the solution of the inequality $2x^2 + 4x + 2 > 0$. (show the solution in the graph)
4th task : What is the solution of the inequality $-x^2 - x - 1 \geq 0$. (show the solution in the graph)
5th task : What is the solution of the inequality $x^2 - 4 > 0$. (show the solution in the graph)
6th task : What is the solution of the inequality $-2x^2 \leq 0$. (show the solution in the graph)


f
A
B
h
d
g
a = 1
b = -4
c = 3

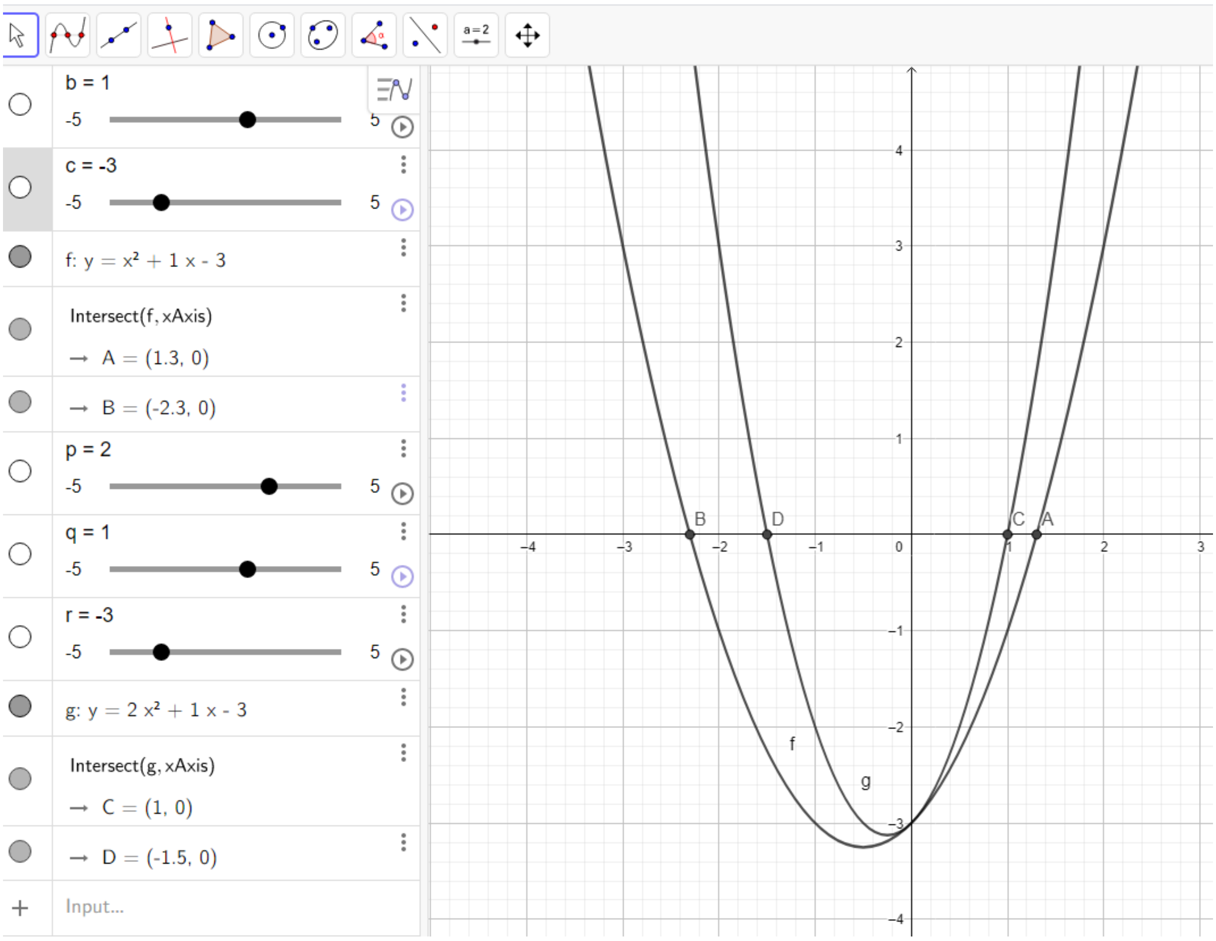
EUROGEBRA WORKSHEET No

TITLE : Roots of quadratics

MENU	TOOL	PROCESS STEPS
		In the input bar enter y=x^2+bx+c
	Find the roots of an equation	Click on the roots button
		Change the values of b and c and answer the questions below
		In the input bar enter y=px^2+qx+r
	Find the roots of an equation	Click on the roots button

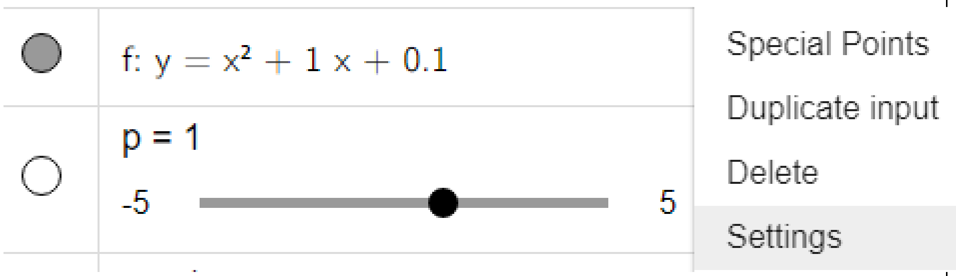
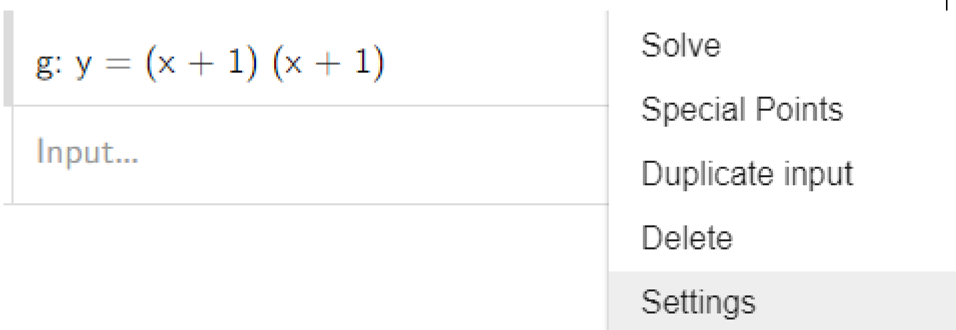
Questions:

1. Add the roots of the equation. Do you notice a link between the roots and b? Add the roots together and negate your answer. This will be the value of b.
2. Multiply the roots together. Do you notice a link between the roots and c? Multiply the roots together and this will be the value of c.
3. Change values of b and c. Are the links still valid? Yes
4. Generalise your observations. The sum of the roots equals -b and the product of the roots equals c
5. Can you see the connections when you use y=px^2+qx+r ? Generalise your observations The sum of the roots equals –q/p whilst the product of the roots equals r/p



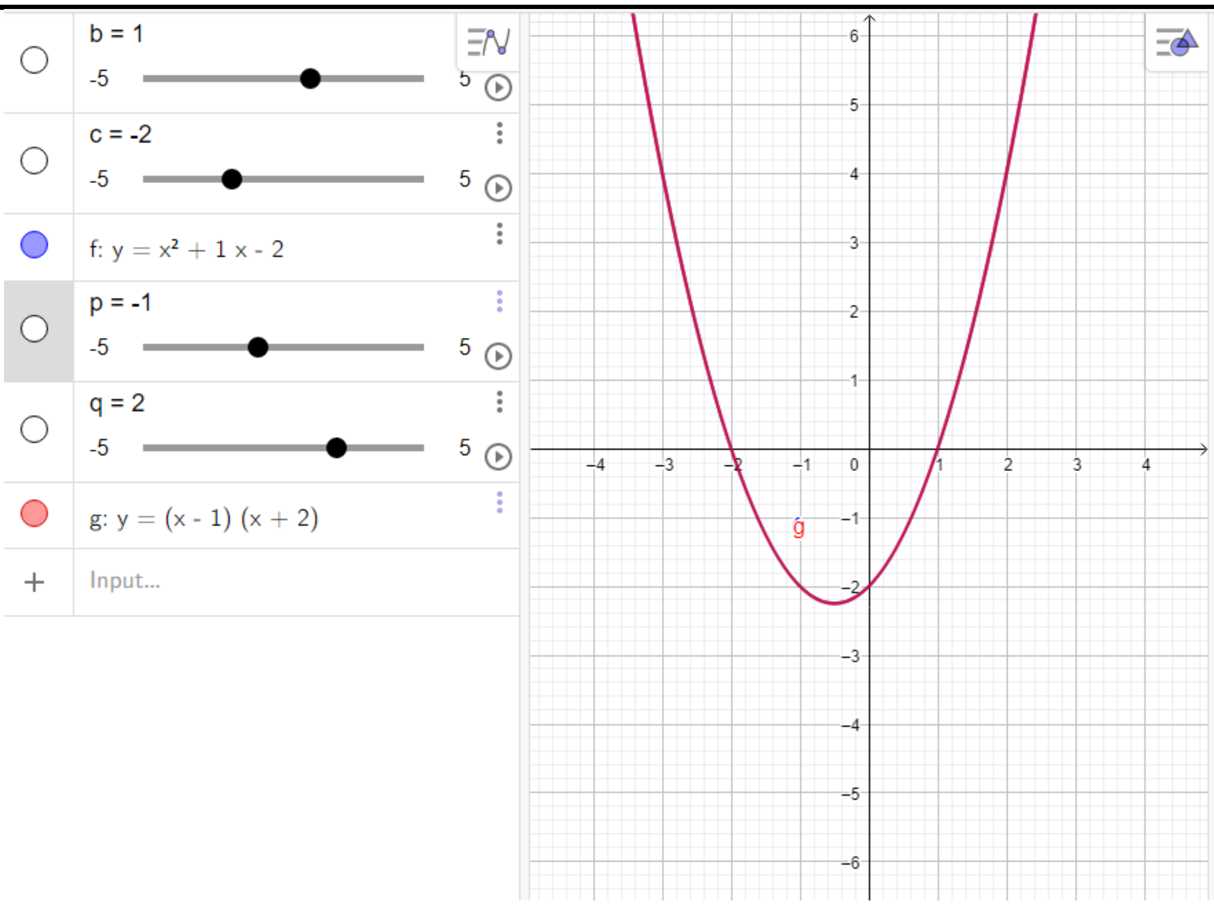
EUROGEBRA WORKSHEET

ROOTS OF FUNCTIONS

MENU	TOOL	PROCESS STEPS
		In the input bar enter y=x^2+bx +c
		Change the colour of the function by clicking on the three dots and going to settings
		In the input bar enter y=(x+p)(x+ q)
		Change the colour of the function by clicking on the three dots and going to settings

Questions:

1. Set b=1 and c=-2 by moving the slider. What is the equation of the function?$y=x^2+1x-2$
2. Set p=-1 and q=2 by moving the slider. What is the equation of the function?$y=(x-1)(x+2)$
3. The two graphs will now coincide. What does this tell you about the two equations?They are the same. When you expand the brackets the equation will be the same as 1.
4. The roots of a quadratic equation are where the graph crosses the x-axis. This gives a y value of 0. How is this linked to the values of p and q above? If you know where they cross the x axis you can work out p and q. This will give a y value of 0.
5. Is there a relationship so that two graphs will always be the same even though you change the values?Yes – if you know where they cross the x axis you can work out p and q. You can then expand to find the equation in the form x^2+bx+c.
6. Can you have a quadratic equation without any roots? Yes – the graph would not cross the x – axis based on the previous definition
7. The answer to question 6 is no. How can this be true?I did not realise you can use complex numbers to represent roots of an equation.


b = 1
-5
5
c = -2
-5
5
f: y = x² + 1 x - 2
p = -1
-5
5
q = 2
-5
5
g: y = (x - 1) (x + 2)
Input...
g

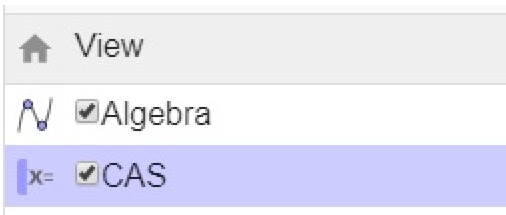
EUROGEBRA WORKSHEET

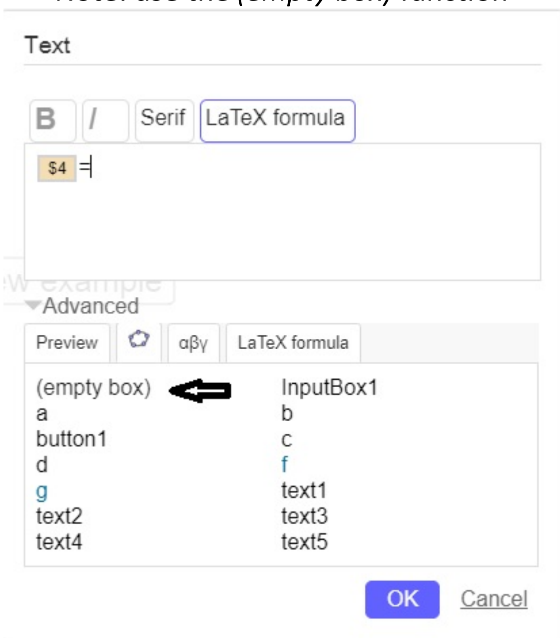
SHOW THE SOLUTIONS TO A QUADRATIC INEQUALITY

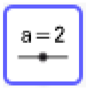
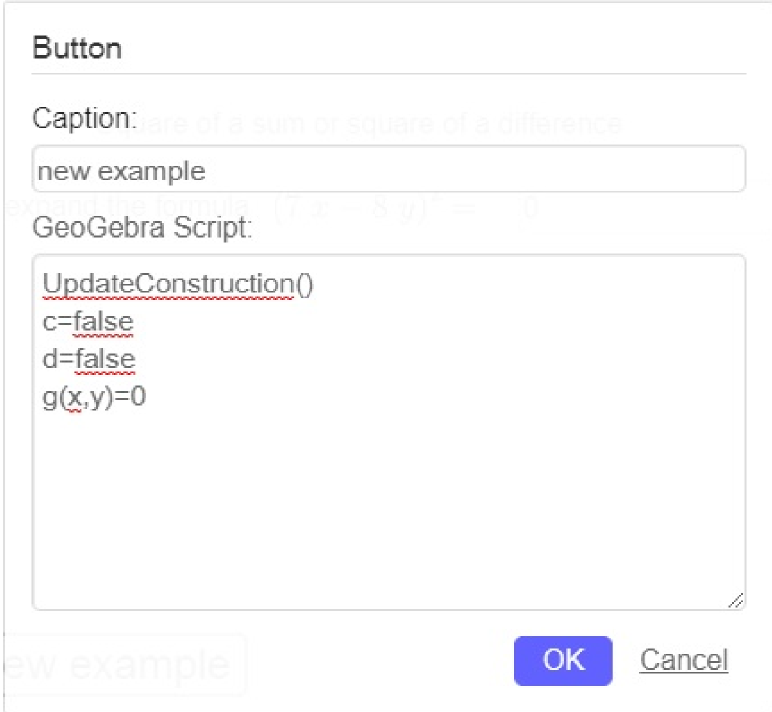
MENU	TOOL	PROCESS STEPS
a = 2	a = 2 Slider	**Create a slider for the variable "a" between -5 and 5.**
a = 2	a = 2 Slider	**Create a slider for the variable "b" between -5 and 5.**
+	Input...	**Input the equation "y = (x – a)(x – b)".**
+	Input...	**Input the inequality"0> (x – a)(x – b)".**
+	Input...	**Input the inequality "0 < (x – a)(x – b)".**

EUROGEBRA WORKSHEET

SHORT MULTIPLICATION FORMULAS

MENU	TOOL	PROCESS STEPS
		In the *Graphics* view hide the grid and both axes
		Open the *CAS* panel 
		In the *CAS* panel type in: *a: = RandomBetween(1,10)*
		In the *CAS* panel type in: *b: = RandomElement({1,2,3,4,5,6,7,8,9,10,-1,-2,-3,-4,-5,-6,-7,-8,-9,-10})*
		In the *CAS* panel type in: *GCD(a,b)* Note: GCD - greatest common divisor
		In the *CAS* panel type in: Factorise$\left(\left(\frac{a}{\$3}x+\frac{b}{\$3}y\right)^2\right)$
		In the *CAS* panel type in: *f(x,y): =$4*
		In the *CAS* panel type in: *G(x,y): = 0*
		In the *CAS* panel type in: *Expand($4)*

	ABC Text	Insert text: *Square of a sum or square of a difference*
	ABC Text	Insert text: *Expand the formula*
	a=1 Input Box	Create an input box *Caption: formula* *Linked object: g(x,y)=0* Hide the label.
		In the *Input bar* type in: *text3 = if(f≠g,"wrong","good job!")*
	ABC Text	Insert a dynamic text (text4) *$4=* Note: use the *(empty box)* function 
	ABC Text	Insert a dynamic text (text5) $4=$7 Note: use the *(empty box)* function again
	Check Box	Insert check box *c*: *Caption: check* *Object: text3*
	Check Box	Insert check box *d*: *Caption: Show correct answer* *Object: text5*

<table>
<tr><td></td><td></td><td>Go to Settings of this object->Advanced,
in Condition to show object type in:
$f \neq g \wedge c$</td></tr>
<tr><td></td><td>OK Button</td><td>Insert a button:
Caption: new example
GeoGebra script: UpdateConstruction()
c=false
d=false
g(x,y)=0

</td></tr>
</table>

End result: